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RECOGNITION OF CHARACTERISTICALLY SIMPLE GROUP $A_5 \times A_5$ BY CHARACTER DEGREE GRAPH AND ORDER

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Abstract. The character degree graph of a finite group G is the graph whose vertices are the prime divisors of the irreducible character degrees of G and two vertices p and q are joined by an edge if pq divides some irreducible character degree of G. It is proved that some simple groups are uniquely determined by their orders and their character degree graphs. But since the character degree graphs of the characteristically simple groups are complete, there are very narrow class of characteristically simple groups which are characterizable by this method.

We prove that the characteristically simple group $A_5 \times A_5$ is uniquely determined by its order and its character degree graph. We note that this is the first example of a non simple group which is determined by order and character degree graph. As a consequence of our result we conclude that $A_5 \times A_5$ is uniquely determined by its complex group algebra.

Keywords: character degree graph; irreducible character; characteristically simple group; complex group algebra

MSC 2010: 20D60, 20D05, 20D08, 20C15

1. Introduction and Preliminary results

Let G be a finite group, $\mathrm{Irr}(G)$ be the set of irreducible characters of G, and denote by $\mathrm{cd}(G)$ the set of irreducible character degrees of G.

One of the main questions in the representation theory is the relation between the irreducible character degrees of G and the structure of G. In [1], Problem 2^*, Brauer asked whether two groups G and H are isomorphic given that two group algebras $\mathbb{F}G$ and $\mathbb{F}H$ are isomorphic for all fields $\mathbb{F}$. This is false in general. In fact, Dade in [3] constructed two non-isomorphic metabelian groups G and H such that $\mathbb{F}G \cong \mathbb{F}H$ for all fields $\mathbb{F}$. Kimmerle in [16] proved that if G is a group and H is a nonabelian simple group such that $\mathbb{F}G \cong \mathbb{F}H$ for all fields $\mathbb{F}$, then $G \cong H$. In [23], Tong-Viet

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proved that each classical simple group is uniquely determined by its complex group algebra. Also he posed the following question:

Question. Which groups can be uniquely determined by the structure of their complex group algebras?

It was shown in [20], [21] that the symmetric groups are uniquely determined by the structure of their complex group algebras. In [16], [19], [22], [24] it is proved that all nonabelian simple groups are uniquely determined by the structure of their complex group algebras. We note that abelian groups are not determined by the structure of their complex group algebras. In fact, the complex group algebras of any two abelian groups of the same orders are isomorphic. There are also examples of nonabelian p-groups with isomorphic complex group algebras, for example the dihedral group of order 8 and the quaternion group of order 8. In [10], [11] and [13] it is proved that some extensions of $\mathrm{PSL}(2,q)$ are uniquely determined by their complex group algebras.

A finite group G is called a K_3-group if $|G|$ has exactly three distinct prime divisors. Chen et al. in [26] and [27] proved that all simple K_3-groups and the Mathieu groups are uniquely determined by their orders and one or both of their largest and the second largest irreducible character degrees.

The *character degree graph* of G, which is denoted by $\Gamma(G)$, is defined as follows: the vertices of this graph are the prime divisors of the irreducible character degrees of the group G and two distinct vertices p_1 and p_2 are joined by an edge if there exists an irreducible character degree of G which is divisible by p_1p_2. This graph was introduced in [18] and studied by many authors (see [17], [25]).

Let p be an odd prime number. In [9] the authors proved that the simple group $\mathrm{PSL}(2,p)$ is uniquely determined by its order and its largest and the second largest irreducible character degrees. In [14] it is proved that the simple group $\mathrm{PSL}(2,p^2)$ is uniquely determined by its character degree graph and its order.

In [12], [15] it is proved that some simple groups are uniquely determined by their orders and their character degree graphs. In this paper, as the first example, we give a characteristically simple group which is uniquely determined by its order and its character degree graph.

If $N \trianglelefteq G$ and $\theta \in \mathrm{Irr}(N)$, then the inertia group of θ in G is $I_G(\theta) = \{g \in G\colon \theta^g = \theta\}$. If the character $\chi = \sum\limits_{i=1}^{k} e_i\chi_i$, where for each $1 \leqslant i \leqslant k$, $\chi_i \in \mathrm{Irr}(G)$ and e_i is a natural number, then each χ_i is called an irreducible constituent of χ.

Lemma 1.1 (Itô's Theorem, see [6], Theorem 6.15)**.** *Let $A \trianglelefteq G$ be abelian. Then $\chi(1)$ divides $|G:A|$ for all $\chi \in \mathrm{Irr}(G)$.*

Lemma 1.2 ([6], Theorems 6.2, 6.8, 11.29)**.** *Let* $N \trianglelefteq G$ *and let* $\chi \in \mathrm{Irr(G)}$. *Let* θ *be an irreducible constituent of* χ_N *and suppose* $\theta_1 = \theta, \ldots, \theta_t$ *are the distinct conjugates of* θ *in* G. *Then* $\chi_N = e \sum_{i=1}^{t} \theta_i$, *where* $e = [\chi_N, \theta]$ *and* $t = |G : I_G(\theta)|$. *Also* $\theta(1) \mid \chi(1)$ *and* $\chi(1)/\theta(1) \mid |G : N|$.

Lemma 1.3 (Itô-Michler Theorem, see [5])**.** *Let* $\varrho(G)$ *be the set of all prime divisors of the elements of* $\mathrm{cd}(G)$. *Then* $p \notin \varrho(G)$ *if and only if* G *has a normal abelian Sylow* p*-subgroup.*

Lemma 1.4 ([26], Lemma 1)**.** *Let* G *be a nonsolvable group. Then* G *has a normal series* $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ *such that* K/H *is a direct product of isomorphic nonabelian simple groups and* $|G/K| \mid |\mathrm{Out}(K/H)|$.

Lemma 1.5 (Gallagher's Theorem, see [6], Corollary 6.17)**.** *Let* $N \trianglelefteq G$ *and let* $\chi \in \mathrm{Irr}(G)$ *be such that* $\chi_N = \theta \in \mathrm{Irr}(N)$. *Then the characters* $\beta\chi$ *for* $\beta \in \mathrm{Irr}(G/N)$ *are irreducible distinct for distinct* β *and all of the irreducible constituents of* θ^G.

Lemma 1.6 ([27], Lemma 2)**.** *Let* G *be a finite solvable group of order* $p_1^{\alpha_1} p_2^{\alpha_2} \ldots \times p_n^{\alpha_n}$, *where* $p_1, p_2, \ldots, p_n$ *are distinct primes. If* $kp_n + 1 \nmid p_i^{\alpha_i}$ *for each* $i \leqslant n-1$ *and* $k > 0$, *then the Sylow* p_n*-subgroup is normal in* G.

Lemma 1.7 ([8], Theorem 3.1)**.** *Let* G *be a finite group and* K *any normal subgroup. Set* $H = G/K$. *Then* $|M(H)|$ *divides* $|M(G)| \cdot |G' \cap K|$, *where* $|M(G)|$ *is the Schur multiplier of* G.

Lemma 1.8 ([8], Theorem 3.2)**.** *Let* G *be a finite group and* B *a central subgroup. Set* $A = G/B$. *Then* $|M(G)| \cdot |G' \cap B|$ *divides* $|M(A)| \cdot |M(B)| \cdot |A \otimes B|$.

If n is an integer and r is a prime number, then we write $r^\alpha \parallel n$ when $r^\alpha \mid n$ but $r^{\alpha+1} \nmid n$. Also if r is a prime number, we denote by $\mathrm{Syl}_r(G)$ the set of Sylow r-subgroups of G and we denote by $n_r(G)$ the number of elements of $\mathrm{Syl}_r(G)$. If H is a subgroup of G, then H_G, the core of H in G, is the largest normal subgroup of G that is contained in H. If H is a characteristic subgroup of G, we write H ch G. All other notations are standard and we refer to [2].

2. Main results

In this section we prove that the characteristically simple group $A_5 \times A_5$ is uniquely determined by its order and its character degree graph.

Theorem 2.1. *Let G be a finite group such that $\Gamma(G) = \Gamma(A_5 \times A_5)$ and $|G| = 3600$. Then $G \cong A_5 \times A_5$. In other words, $A_5 \times A_5$ is characterizable by its order and its character degree graph.*

Proof. First we prove that the finite group G is not a solvable group. On the contrary let G be a solvable group of order $2^4 3^2 5^2$ and N be a normal minimal subgroup of G. In the sequel we consider the following cases:

(i) Let N be a 2-elementary abelian group. Hence $|N| = 2^i$, where $1 \leqslant i \leqslant 3$, by Itô-Michler theorem, since 2 is a vertex of $\Gamma(G)$. So $|G/N| = 2^{4-i}3^2 5^2$ and by Lemma 1.6 it follows that $Q/N \in \mathrm{Syl}_5(G/N)$ is a normal subgroup of G/N. Therefore $Q \lhd G$ and $|Q| = 2^i 5^2$. Again, since $i \leqslant 3$, by Lemma 1.6 we get that $P \in \mathrm{Syl}_5(Q)$ is a normal subgroup of G which is a contradiction by Itô-Michler theorem since $5 \in \varrho(G)$.

(ii) Let N be a 5-elementary abelian group. By assumptions we get that $|N| = 5$. Now suppose that L/N is a normal minimal subgroup of G/N. Obviously L/N is not a 5-group. In the sequel we consider two subcases:

(a) If L/N is a 2-group, then there exists $1 \leqslant i \leqslant 4$ such that $|L| = 2^i 5$ and $|G/L| = 2^{4-i}3^2 5$. By Lemma 1.6 we get that $Q/L \lhd G/L$, where $Q/L \in \mathrm{Syl}_5(G/L)$. Hence $|Q| = 2^i 5^2$ and $Q \lhd G$. Now P, a Sylow 5-subgroup of Q, is not a normal subgroup of Q and so $i = 4$ and $n_5(Q) = |Q : N_Q(P)| = 16$. Therefore $P = N_Q(P)$ and so $P \subseteq Z(N_Q(P))$. Then by Burnside p-complement theorem we get that P has a normal 5-complement R in Q, which is the Sylow 2-subgroup of G. We note that L/N is a 2-elementary abelian group of order 2^4. Since $L = RN$, we get that $L/N \cong RN/N \cong R$, and so R is normal and abelian, which is a contradiction.

(b) If L/N is a 3-group, then $|L| = 3^j 5$, where $1 \leqslant j \leqslant 2$. By Lemma 1.6 we get that $|L| = 15$ and $L \lhd G$. Hence T/L, a normal minimal subgroup of G/L, is a 2-elementary abelian group and $|T/L| = 2^i$, where $1 \leqslant i \leqslant 4$. Therefore $|T| = 2^i \times 15$ and $|G/T| = 2^{4-i} \times 15$. Let $Q/T \in \mathrm{Syl}_5(G/T)$. Then $Q \lhd G$ and $|Q| = 2^i 5^2 3$. Since a Sylow 5-subgroup of G is not a normal subgroup, we get that $|Q| = 2^4 5^2 3$. Let K be a Hall subgroup of Q such that $|Q : K| = 3$. Then $Q/K_Q \hookrightarrow S_3$, where $K_Q = \mathrm{core}_Q(K)$, and so $|K_Q| = 2^3 5^2$ or $|K_Q| = 2^4 5^2$. If $|K_Q| = 2^3 5^2$, then the Sylow 5-subgroup of G is a normal subgroup of G, which is a contradiction. If $|K_Q| = 2^4 5^2$, then $n_5(K_Q) = 16$ and so $P \subseteq Z(N_{K_Q}(P))$, where $P \in \mathrm{Syl}_5(K_Q)$. Then by Burnside p-complement theorem we get that P has a normal

5-complement R in K_Q, which is the Sylow 2-subgroup of G. Now $L/N \cong R$ implies that R is normal and abelian, which is a contradiction.

(iii) Let N be a 3-elementary abelian group. Then $|N| = 3$ and $|G/N| = 2^4 5^2 3$. By considering L/N, where $|G/N : L/N| = 3$, we get a normal subgroup M of G such that $|M| = 2^4 5^2 3$ or $|M| = 2^3 5^2 3$. In case (ii) we proved that $|M| = 2^4 5^2 3$ is impossible and so $|M| = 2^3 5^2 3$. Then the Sylow 5-subgroup of M is a normal subgroup of M, which is a contradiction.

Therefore G is not a solvable group and so G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that K/H is isomorphic to a direct product of m copies of a simple group S and $|G/K| \mid |\mathrm{Out}(K/H)|$. Using [2] we get that $K/H \cong A_5, A_6$ or $A_5 \times A_5$. Now we consider each possibility for K/H separately.

Step 1. Let $K/H \cong A_5$. Then $|H| = 30$ or $|H| = 60$.

(1.1) Let $|H| = 30$ and $|G/K| = 2$.

Each finite group of order 30 is solvable and we know that H has a normal subgroup T of order 15 which is cyclic and $T \lhd K$. Therefore $n_3(H) = 1$ and $n_5(H) = 1$. Since the Sylow 3-subgroup and the Sylow 5-subgroup of H are normal and abelian, by Lemma 1.3 we get that $3 \notin \varrho(H)$, $5 \notin \varrho(H)$. Therefore $\varrho(H) \subseteq \{2\}$. We claim that in this case there exists no $\chi \in \mathrm{Irr}(G)$ such that $15 \mid \chi(1)$. On the contrary, let $\chi \in \mathrm{Irr}(G)$ and $15 \mid \chi(1)$. Since $K \lhd G$ and $|G : K| = 2$, there exists $\eta \in \mathrm{Irr}(K)$ such that $15 \mid \eta(1)$. Using Lemma 1.2, there exists $\theta \in \mathrm{Irr}(T)$ such that

$$\eta_T = e \sum_{i=1}^{t} \theta_i,$$

where $t = |K : I_K(\theta)|$ and $\theta_1, \ldots, \theta_t$ are all conjugates of θ in K. Also by assumptions we know that $9 \mid |C_G(T)|$ and $25 \mid |C_G(T)|$. Therefore $t = |K : I_K(\theta)|$ is a divisor of 8. On the other hand, $15 \mid \eta(1)$ and so $15 \mid et$ which implies that $15 \mid e$. We know that $15^2 \leqslant e^2 \leqslant |K : T| = 8 \times 15$, which is a contradiction. Therefore this case is impossible.

(1.2) *Let* $|H| = 60$ *and so* $G = K$.

First suppose that H is a solvable group. Again we prove that there exists no irreducible character χ of G such that $15 \mid \chi(1)$.

For this purpose let $X = O^2(G)$. Then since $X \lhd G$ and $|G : X| = 2^\alpha$ for some $\alpha \geqslant 0$, by Clifford's theorem we get that it is enough to prove that there is no $\theta \in \mathrm{Irr}(X)$ such that $15 \mid \theta(1)$. On the contrary, let there exist $\theta \in \mathrm{Irr}(X)$ and $15 \mid \theta(1)$. If R is a Sylow 5-subgroup of X or a Sylow 3-subgroup of X, then R is abelian and by transfer (see [7], Theorem 5.3) we get that $R \cap X' \cap Z(X) = 1$. On the other hand, $HX/H \lhd G/H$ and since $G/H \cong A_5$ and $X \nsubseteq H$, we get that $G = HX$.

By assumptions, G/X is a 2-group and since $G/X \cong HX/X \cong H/(H \cap X)$, we get that $[G : X] \mid 4$. Let L be the terminal member of the derived series of G. Then $L' = L$ and since LX/X is abelian, we get that $L \leqslant X$. Therefore $L \leqslant X'$. Since H is a solvable group, by Lemma 1.6 we get that the Sylow 5-subgroup of H is a normal subgroup of H.

In the sequel we consider two subcases. First suppose that the Sylow 3-subgroup of H is a normal subgroup of H. Then H has a cyclic subgroup M of order 15, which is normal in G. Also $M \leqslant Z(X)$ since $\mathrm{Aut}(M)$ is a group of order 8 and $M \lhd G$. Therefore $M \cap X' = 1$ and so $M \cap L = 1$. Hence $|L| \leqslant 240$. By the table in [4], Section 5.4 we get that $L \cong A_5$ or $L \cong \mathrm{SL}(2,5)$. This implies that $X \cong LM \cong L \times M$ has no irreducible character χ such that $15 \mid \chi(1)$ and since $|G : LM| \mid 4$, we get that $3 \nsim 5$ in $\Gamma(G)$, a contradiction.

Hence Q, a Sylow 3-subgroup of H, is not a normal subgroup of H. By assumptions, $P \lhd G$ and so $|G : C_G(P)| \mid 4$. Therefore $X \subseteq C_G(P)$ and so $P \leqslant Z(X)$. By transfer we get that $P \cap X' = 1$. Therefore $P \cap L = 1$ and so $|L| \mid 720$ and $LH \lhd G$, which implies that $LH = H$ or $LH = G$. Since H is solvable and $L' = L$, we get that $L \nleqslant H$. Therefore $LH = G$ and $A_5 \cong G/H = LH/H \cong L/(L \cap H)$. Since $|L| \mid 720$, by the table in [4], Section 5.4 we get that L is isomorphic to $A_5, \mathrm{SL}(2,5)$ or A_6. Hence $L \cong A_5$ or $L \cong \mathrm{SL}(2,5)$. If $L \cong A_5$, then $|L \cap H| = 1$ and $G \cong L \times H \cong A_5 \times H$, which is a contradiction. If $L \cong \mathrm{SL}(2,5)$, then $|L \cap H| = 2$ and $L \cap H \lhd H$, which implies that $L \cap H \subseteq Z(H)$, and this is a contradiction.

Therefore H is not solvable and so $H \cong A_5$ and $G/H \cong A_5$. Since H is a non-abelian simple group, we get that $H \cap C_G(H) = 1$. Also $HC_G(H) \cong H \times C_G(H)$ and $C_G(H) \cong HC_G(H)/H \lhd G/H \cong A_5$. Therefore $G \cong H \times C_G(H) \cong A_5 \times A_5$.

Step 2. If $K/H \cong A_6$, then $|H| = 5$ or $|H| = 10$.

If $|H| = 5$, then $|G/K| = 2$. Hence $K/C_K(H) \hookrightarrow \mathrm{Aut}(H)$, which implies that $H \leqslant Z(K)$. Also the Schur multiplier of A_6 is 6, which implies that $K \cong A_6 \times \mathbb{Z}_5$. Since $|G : K| = 2$, we get that 3 and 5 are not adjacent in $\Gamma(G)$, which is a contradiction.

If $|H| = 10$, then $G = K$ and $H \cong D_{10}$ or $H \cong Z_{10}$.

If $H \cong D_{10}$, then $Z(H) = 1$ and so $H \cap C_G(H) = 1$. On the other hand, $C_G(H) \cong HC_G(H)/H \lhd G/H$ and $G/H \cong A_6$, which implies that $C_G(H) \cong A_6$ and $G = HC_G(H)$. Therefore $G \cong H \times C_G(H) \cong D_{10} \times A_6$. Hence $\Gamma(G)$ is not complete and we get a contradiction.

If $H \cong \mathbb{Z}_{10}$, then obviously $H \leqslant C_G(H)$ and $G/C_G(H) \hookrightarrow \mathrm{Aut}(\mathbb{Z}_{10})$. Since $G/H \cong A_6$, we get that $C_G(H) \cong G$. Hence $H \leqslant Z(G)$. Let A be the Sylow 5-subgroup of H. Then $(G/A)/(H/A) \cong A_6$ and $|H/A| = 2$. Hence G/A is the central extension of $\mathbb{Z}_2$ by A_6, which implies that $G/A = 2 \cdot A_6 = \mathrm{SL}(2,9)$ or $G/A = \mathbb{Z}_2 \times A_6$.

If $G/A \cong \mathrm{SL}(2,9)$, then by Lemmas 1.7 and 1.8, $6 = |M(G/A)| \mid |M(G)| \cdot |G' \cap A|$ and $|M(G)| \cdot |G' \cap A|$ is a divisor of $|M(G/A)| \cdot |M(A)| \cdot |A \otimes \mathrm{SL}(2,9)|$. Now since $\mathrm{SL}(2,9)$ is a perfect group, we get that $G' \cap A = 1$ and $M(G) = 6$. On the other hand, $G/A \cong \mathrm{SL}(2,9)$, and so $G'A/A \cong \mathrm{SL}(2,9)$, which implies that $G \cong G'A \cong G' \times A$. We know that $G' \cong G'A/A \cong \mathrm{SL}(2,9)$, and so $G \cong \mathrm{SL}(2,9) \times \mathbb{Z}_5$, which is a contradiction since $\Gamma(G)$ is not a complete graph.

Finally, if $G/A \cong \mathbb{Z}_2 \times A_6$, then there exists a normal subgroup T/A of G/A such that $|G : T| = 2$ and $T/A \cong A_6$.

Similarly to the previous discussion we get that $T \cong \mathbb{Z}_5 \times A_6$ since $|M(A_6)| = 6$ and so we get a contradiction.

Step 3. If $K/H \cong A_5 \times A_5$, then obviously $G \cong A_5 \times A_5$ and we get the result. $\square$

Remark. As a consequence of our result it is proved that the charactertically simple group $A_5 \times A_5$ is uniquely determined by its complex group algebra.

Remark. If M is any group of order 36, we see that $M \times A_5 \times A_5$ has the same order as $A_6 \times A_6$. Also the character degree graph of this group is the complete graph on the vertex set $\{2,3,5\}$. Therefore $A_6 \times A_6$ is not charactertizable by order and the character degree graph. Similarly, it is not difficult to see that for $5 < n < 20$ there exists no n such that $A_n \times A_n$ is characterizable by order and the character degree graph.

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