Czechoslovak Mathematical Journal, 68 (143) (2018), 1033–1050

ON SOME NEW SHARP ESTIMATES IN ANALYTIC HERZ-TYPE FUNCTION SPACES IN TUBULAR DOMAINS OVER SYMMETRIC CONES

Romi F. Shamoyan, Bryansk, Olivera Mihić, Belgrade

Received February 11, 2017. Published online April 20, 2018.

Abstract. We obtain new sharp embedding theorems for mixed-norm Herz-type analytic spaces in tubular domains over symmetric cones. These results enlarge the list of recent sharp theorems in analytic spaces obtained by Nana and Sehba (2015).

Keywords: analytic function; tubular domain; embedding theorem

MSC 2010: 42B15, 42B30

1. Introduction

The main goal of this paper is to enlarge the list of sharp embedding theorems for analytic function spaces in tubular domains over symmetric cones obtained recently in [6], [16] and [17].

Various embeddings of analytic function spaces in various types of domains in higher dimension and their numerous applications were under intensive attention by various authors during last decades (see, for example, [1], [2], [6], [8], [15], [16], [17], [25] and many references therein). In this paper we will turn to the study of certain new sharp embedding theorems for some new mixed norm analytic classes in tubular domains over symmetric cones in $\mathbb{C}^n$. This paper can be considered as a continuation of recent investigations in this direction, namely other new sharp results of such type in tube domains over symmetric cones have been given recently in [6], [16] and [17]. Related new sharp embedding theorems of such type in other

The first author has been supported by the Ministry of Education and Science of the Russian Federation (grant 1.1704.2014J), the second author has been supported by MNTR Serbia, Project 174017.

DOI: 10.21136/CMJ.2018.0059-17

general (bounded) pseudoconvex domains with proofs based on the same ideas have been provided recently in [22]. In this paper we find complete analogues of some theorems from [14], where they can be seen in context of less general unit ball in tube domains over symmetric cones. For proving the estimates and embedding theorems in tubular domains over symmetric cones we heavily use the same technique as has been developed recently in [1], [2] in pseudoconvex domains, but in context of tube domains. In [16] and [17] in tube domains and in [1] and [2] in pseudoconvex domains, a crucial estimate from below of Bergman kernel on Bergman ball and Kobayashi balls appeared and was used and this estimate will often be used by us in the proofs below. Similar ideas were used in context of a unit ball and pseudoconvex domains in [14] and [22]. For similar sharp embedding results based on similar ideas in bounded symmetric domains and their direct generalizations—minimal bounded homogeneous domains, we refer the reader to a series of recent subtle results of Yamaji (see [23], [24] and various references therein). Note that pseudoconvex domains are not symmetric, tubular domains are not bounded. Minimal bounded homogeneous domains serve as direct extensions of bounded symmetric domains (see [23], [24] and references therein). The motivation of this paper is to provide complete analogues of some of our recent sharp results for so-called Herz-type analytic spaces in tube domains over symetric cones. We refer to [22], where our theorems can be seen in bounded pseudoconvex domains and minimal homogeneous domains; new related results on sharp embeddings on other domains were also discussed in that paper. Since till now there are only several sharp embedding theorems in analytic function spaces in domains with very complex structure in higher dimension, we found these reasons enough to present a new paper. It is well known that applications of such type of results are numerous. In various embeddings theorems for analytic mixed norm function spaces in tubular domains over symmetric cones as well as in other domains, the so-called Carleson type measures constantly appear. We turn to some history related to this problem starting from the simplest domain—the unit disk. Carleson measures were introduced by Carleson (see [7]) in his solution of the corona problem in the unit disk of the complex plane, and since then they have become an important tool in analysis, and an interesting object of study per se. Let A be a Banach space of analytic functions on a domain $D \subset \mathbb{C}^n$. Given $p \geqslant 1$, a finite positive Borel measure μ on D is a *Carleson measure* of A (for p) if there is a continuous inclusion $A \hookrightarrow L^p(\mu)$, that is, there exists a constant $C > 0$ such that

$$\int_D |f|^p \,\mathrm{d}\mu \leqslant C\|f\|_A^p \quad \forall\, f \in A.$$

A finite positive Borel measure μ is a Carleson measure of Hardy space $H^p(\mathbb{D})$, where $\mathbb{D}$ is the unit disk in $\mathbb{C}$ (see [19]) if and only if there exists a constant $C > 0$

such that $\mu(S_{\theta_0,h}) \leqslant Ch$ for all sets

$$S_{\theta_0,h} = \{r\mathrm{e}^{\mathrm{i}\theta} \in \mathbb{D}\colon\ 1-h \leqslant r < 1,\ |\theta - \theta_0| < h\}$$

(see also [10], [19]). The set of Carleson measures of $H^p(\mathbb{D})$ does not depend on p.

In [12] the authors obtained a similar description for the Carleson measures of the Bergman spaces $A^p(\mathbb{D})$ (see also [18] and [19] for such type of result); it was also obtained in terms of the special sets $S_{\theta_0,h}$. In [8] and [9] the authors characterized Carleson measures for Bergman spaces in the unit ball $B^n \subset \mathbb{C}^n$, and Cima and Mercer (see [8], [9]) found the description of Carleson measures of Bergman spaces in strongly pseudoconvex domains, showing in particular that the set of Carleson measures of $A^p(D)$ is independent of $p \geqslant 1$. We turn to more details. In [8] and [9] a characterization of Carleson measures of Bergman spaces was formulated in terms of more general sets than $S_{\theta_0,h}$. We will use the one expressed via the intrinsic Kobayashi geometry of the domain. Let $z_0 \in D$ and $0 < r < 1$, let $B_D(z_0,r)$ denote the ball of center z_0 and radius $\frac{1}{2}(\log(1+r) - \log(1-r))$ for the Kobayashi distance k_D of D (that is, of radius r with respect to the pseudohyperbolic distance $\varrho = \tanh(k_D)$). It is known for D strongly pseudoconvex (see [1], [2]) that a finite positive measure μ is a Carleson measure of $A^p(D)$ for p if and only if for some (and hence all) $0 < r < 1$ there is a constant $C_r > 0$ such that

$$\mu(B_D(z_0,r)) \leqslant C_r \nu(B_D(z_0,r)) \quad \forall\, z_0 \in D,$$

where ν is a standard Lebegue's measure on a domain. (The proof of this in [2] relied on Cima and Mercer's characterization, see [8].)

We say that a finite positive Borel measure μ is a (geometric) θ-*Carleson measure* if for some (and hence all) $0 < r < 1$ there is a constant $C_r > 0$ such that

$$\mu(B_D(z_0,r)) \leqslant c_r \nu(B_D(z_0,r))^\theta \quad \forall\, z_0 \in D.$$

Note that 1-Carleson measures are the usual Carleson measures of $A^p(D)$, and we know that θ-Carleson measures are exactly the Carleson measures of weighted Bergman spaces (see [1], [2]). Note also that when $D = B^n$, a q-Carleson measure in the sense of [13], [15] is a $(1 + q(n+1)^{-1})$-Carleson measure in our sense. We refer the reader to [3], [4], [5], [15], [25] and several references therein for various (not only sharp) embedding theorems and related results in the case of a ball for analytic Bergman type and Besov type spaces in higher dimension and for various embeddings related to mixed norm spaces of analytic functions of several variables in tubular domains over symmetric cones. In this paper we are however more interested in Carleson type measures for some new analytic Herz-type mixed norm spaces in tubular domains over symmetric cones. Note in addition that the literature concerning various one-dimensional embeddings is very large (see, for example, [15]). In recent papers of Yamaji (see [23], [24] and references therein) new subtle estimates from below for Bergman kernel and weighted Bergman kernel on balls forming r-lattices were provided in context of bounded minimal homogeneous domains. This leads to new sharp embedding theorems for these domains (see [14], [15]). Similarly as in bounded pseudoconvex domains with smooth boundary in tubular domains over symmetric cones, some sharp Carleson type embeddings for Bergman type spaces (or Herz spaces) will be also fully characterized in terms of Carleson type measures of tubular domains (see the definitions of Carleson type measures for these domains below). Throughout this paper constants are denoted by C and C_i, $i = 1, 2, \ldots$, or by C with other indices, they are positive and may not be the same at each occurrence.

2. Preliminaries on geometry of tubular domains over symmetric cones

In this section we provide a chain of facts, properties and estimates on the geometry of tubular domains which we will use in all our proofs below. Practically all of them are taken from recent interesting papers of Bekolle and coauthors, see [3], [4], [5]. In particular, following these papers we provide several results on the boundary behavior of Bergman balls and we formulate a vital submean property for analytic functions in Bergman balls. Note that two estimates are very important for us. The first one is the so-called Forelly-Rudin estimate in tube and the second one is the estimate from below of Bergman kernel on Bergman balls (see the recent papers [16], [17] and also [5]).

If $D = T_\Omega$ is a tube domain over cone, $z_0 \in D$ and $r \in (0, 1)$, we shall denote by $B_D(z_0, r)$ the Bergman ball of center z_0 and radius r.

Let $T_\Omega = V + \mathrm{i}\Omega$ be the tube domain over an irreducible symmetric cone Ω in the complexification $V^{\mathbb{C}}$ of an n-dimensional Euclidean space V. Following the notation of [3] we denote the rank of the cone Ω by r and by Δ the determinant function on V. Letting $V = \mathbb{R}^n$, we have as an example of a symmetric cone on $\mathbb{R}^n$, the Lorentz cone Λ_n defined for $n \geqslant 3$ as

$$\Lambda_n = \{y \in \mathbb{R}^n\colon\ y_1^2 - \ldots - y_n^2 > 0,\ y_1 > 0\}.$$

It is equivalent to the forward light cone given by

$$\{y = (y_1, y_2, y') \in \mathbb{R}^n\colon\ y_1 y_2 - |y'|^2 > 0\}.$$

Light cones have rank 2. The determinant function in this case is given by the Lorentz form $\Delta(y) = y_1^2 - \ldots - y_n^2$ (see, for example, [3]).

$\mathcal{H}(T_\Omega)$ denotes the space of all holomorphic functions on $D = T_\Omega$.

For $\tau \in \mathbb{R}_+$ and the associated determinant function $\Delta(x)$ we set

$$(2.1) \qquad A_\tau^\infty(T_\Omega) = \Big\{F \in \mathcal{H}(T_\Omega)\colon \ \|F\|_{A_\tau^\infty} = \sup_{x+\mathrm{i}y \in T_\Omega} |F(x+\mathrm{i}y)|\Delta^\tau(y) < \infty\Big\}$$

(see [3] and references therein). It can be checked that this is a Banach space.

For $1 \leqslant p, q < \infty$, $\nu \in \mathbb{R}$ and $\nu > nr^{-1} - 1$ we denote by $A_\nu^{p,q}(T_\Omega)$ the mixed-norm weighted Bergman space consisting of analytic functions f in T_Ω that

$$\|F\|_{A_\nu^{p,q}} = \bigg(\int_\Omega \bigg(\int_V |F(x+\mathrm{i}y)|^p \,\mathrm{d}x\bigg)^{q/p} \Delta^\nu(y) \frac{\mathrm{d}y}{\Delta(y)^{n/r}}\bigg)^{1/q} < \infty.$$

This is a Banach space. Replacing simply A by L we will get as usual the corresponding larger space of all measurable functions in tube over symmetric cone with the same quazinorm (see [3], [21]). It is known that the $A_\nu^{p,q}(T_\Omega)$ space is nontrivial if and only if $\nu > nr^{-1} - 1$ (see [3], [21]), and we will assume this everywhere below. When $p = q$, we write (see [3])

$$A_\nu^{p,q}(T_\Omega) = A_\nu^p(T_\Omega).$$

This is the classical weighted Bergman space with usual modification when $p = \infty$.

The (weighted) Bergman projection P_ν is the orthogonal projection from the Hilbert space $L_\nu^2(T_\Omega)$ onto its closed subspace $A_\nu^2(T_\Omega)$ and it is given by the following integral formula (see [3]):

$$(2.2) \qquad P_\nu f(z) = C_\nu \int_{T_\Omega} B_\nu(z,w) f(w) \Delta^{\nu - n/r}(v) \,\mathrm{d}u \,\mathrm{d}v,$$

where $B_\nu(z,w) = C_\nu \Delta^{-(\nu+n/r)}((z-\overline{w})/\mathrm{i})$ is the weighted Bergman reproducing kernel for $A_\nu^2(T_\Omega)$ (see [3]). In the sequel we use constantly the following notations $w = u + \mathrm{i}v \in T_\Omega$ and also $= x + \mathrm{i}y \in T_\Omega$.

Let us first recall the following known basic integrability properties for the determinant function, which appeared already in the definitions above. Below we denote by Δ_s the generalized power function (see [3]).

Lemma 2.1.

(1) *The integral*

$$J_\alpha(y) = \int_{\mathbb{R}^n} \Big|\Delta^{-\alpha}\Big(\frac{x+\mathrm{i}y}{\mathrm{i}}\Big)\Big| \,\mathrm{d}x$$

converges if and only if $\alpha > 2nr^{-1} - 1$. *In that case*

$$J_\alpha(y) = C_\alpha \Delta^{-\alpha+n/r}(y), \quad \alpha \in \mathbb{R},\ y \in \Omega.$$

(2) *Let* $\alpha \in \mathbb{C}^r$ *and* $y \in \Omega$. *For any multi-indices* s *and* β *and* $t \in \Omega$ *the function* $y \mapsto \Delta_\beta(y+t)\Delta_s(y)$ *belongs to* $L^1(\Omega, \mathrm{d}y/\Delta^{n/r}(y))$ *if and only if* $\Re s > g_0$ *and* $\Re(s+\beta) < -g_0^*$. *In that case we have*

$$\int_\Omega \Delta_\beta(y+t)\Delta_s(y)\frac{\mathrm{d}y}{\Delta^{n/r}(y)} = C_{\beta,s}\Delta_{s+\beta}(t).$$

We refer to Corollary 2.18 and Corollary 2.19 of [21] for the proof of the above lemma or to [3]. As a corollary of the one-dimensional version of the second estimate and the first estimate (see, for example, [21]) we obtain the following vital Forelli-Rudin estimate (2.3) which we will often use in the proofs of our main results:

$$\int_{T_\Omega} \Delta^\beta(y)|B_{\alpha+\beta+n/r}(z,w)|\,\mathrm{d}\nu(z) = C\Delta^{-\alpha}(v), \tag{2.3}$$

where $\beta > -1$, $\alpha > nr^{-1} - 1$, $z = x + \mathrm{i}y$, $w = u + \mathrm{i}v$ (see [21]).

In this paper we restrict ourselves to Ω irreducible symmetric cone in the Euclidean vector space $\mathbb{R}^n$ of dimension n, endowed with an inner product for which the cone Ω is self dual. We denote by $D = T_\Omega = \mathbb{R}^n + \mathrm{i}\Omega$ the corresponding tube domain in $\mathbb{C}^n$.

Let $D \Subset \mathbb{C}^n$ be a tube domain within $\mathbb{C}^n$. We shall use the following notations:

- ▷ $\delta\colon D \to \mathbb{R}^+$ will denote the determinant function, that is, $\delta(z) = \Delta(\mathrm{Im}; z)$, $z \in T_\Omega = D$;
- ▷ $\mathrm{d}\nu$ will be the Lebesgue measure on T_Ω;
- ▷ given $1 \leqslant p \leqslant \infty$, the *Bergman space* $A^p(D)$ is the Banach space $L^p(D) \cap H(D)$ endowed with the L^p-norm;
- ▷ $B\colon D \times D \to \mathbb{C}$ will be the Bergman kernel of D, $B = B_{n/r}$.

Let further $\mathrm{d}\nu_t(z) = (\delta(z))^t\,\mathrm{d}\nu(z)$, $t > -1$.

Definition 2.1. Let $D \Subset \mathbb{C}^n$ be a bounded domain and $r > 0$. An r-lattice in D is a sequence $\{a_k\} \subset D$ such that $D = \bigcup\limits_k B_D(a_k, r)$ and there exists $m > 0$ such that any point in D belongs to at most m balls of the form $B_D(a_k, R)$, where $R = \frac{1}{2}(1+r)$.

The existence of r-lattices in tube $D = T_\Omega$ domains is ensured by the following.

Lemma 2.2 ([3], Lemma 2.5). *Let* $D \subset\subset \mathbb{C}^n$ *be a domain. Then for every* $r \in (0,1)$ *there exists an* r*-lattice in* D, *that is, there exist* $m \in N$ *and a sequence* $\{a_k\} \subset D$ *of points such that* $D = \bigcup\limits_{k=0}^{\infty} B_D(a_k, r)$ *and no point of* D *belongs to more than* m *of the balls* $B_D(a_k, R)$, $R = c(r)$, *where* $c(r)$ *is a certain function of* r.

Note that we have

$$\nu_\alpha(B_D(a_k,R)) = \int_{B_D(a_k,R)} \delta^\alpha(z)\,\mathrm{d}\nu(z) = \delta^\alpha(a_k)\nu(B_D(a_k,R))$$
$$= \delta^\alpha(a_k)\int_{B_D(a_k,R)} \mathrm{d}\nu(z), \quad \alpha > -1.$$

This equality follows directly from the properties of r-lattices on Bergman balls we listed already and the definition of weighted Lebesgue's measure.

We will call r-lattice sometimes the family of balls $B_D(a_k,r)$. Dealing with unweighed Bergman kernel ($B = B_{n/r}$) we always assume $|B(z,a_k)| \asymp |B(a_k,a_k)|$ for any $z \in B_D(a_k,r)$, $r \in (0,1)$ (see [3], [4]). Based on the definition of the Bergman kernel (see [3]), it is easy to see this assertion is valid also for all B_t kernels, $t = mnr^{-1}$ if $m \in \mathbb{N}$.

More general version of this assertion can be seen in Corollary 2.1.

We shall use a submean estimate for nonnegative subharmonic functions on Bergman balls on tube $D = T_\Omega$ domains. We denote for simplicity tube domains by D.

Lemma 2.3 ([3]). *Let $D \subset\subset \mathbb{C}^n$ be a tube domain. Given $r \in (0,1)$, set $R = \frac{1}{2}(1+r) \in (0,1)$. Then there exists $C_r > 0$ depending on r such that*

$$\forall\, z_0 \in D,\ \forall\, z \in B_D(z_0,r) \quad \chi(z) \leqslant \frac{C_r}{\nu(B_D(z_0,r))}\int_{B_D(z_0,R)} \chi\,\mathrm{d}\nu$$

for every nonnegative subharmonic function $\chi\colon\ D \to \mathbb{R}^+$.

We will use this lemma for $\chi = |f(z)|^q$, $f \in H(D)$, $q \in (0,\infty)$.

Remark 2.1. Note that from Lemma 2.3 for each $a_0 \in D$ we have

$$\sup_{z\in B_D(a_0,r)} |f(z)|^q \leqslant \frac{C_r}{\nu(B_D(a_0,r))}\int_{B_D(a_0,r)} |f(w)|^q\,\mathrm{d}\nu(w), \quad f \in H(D),\ q \in (0,\infty).$$

We have the following result known as Korányi's lemma.

Lemma 2.4 ([3], [16], [17], [21]). *For every $\delta > 0$ there is a constant $C_\delta > 0$ such that*

$$\Big|\frac{B(\zeta,z)}{B(\zeta,w)} - 1\Big| \leqslant C_\delta d(z,w)$$

for all $\zeta, z, w \in D$ with $d(z,w) \leqslant \delta$, where d is a distance in T_Ω (see [3], [21]).

The following vital for us corollary is a straightforward consequence of [3], [21] and Lemma 2.4 (the estimate from below of Bergman kernel).

First, for $\nu > nr^{-1} - 1$ and $w \in D$, the normalized reproducing kernel is

$$b_\nu(\cdot, w) = \frac{B_\nu(\cdot, w)}{\|B_\nu(\cdot, w)\|_{2,\nu}} = \Delta^{-\nu-n/r}\Big(\frac{\cdot - \overline{w}}{\mathrm{i}}\Big)\Delta^{(\nu+n/r)/2}(\operatorname{Im} w). \tag{2.4}$$

Corollary 2.1 ([16]). *Let $\nu > nr^{-1} - 1$, $\delta > 0$ and $z, w \in D$. There is a positive constant C_δ such that for all $z \in B_\delta(w)$,*

$$V_{\nu-n/r}(B_\delta(w))|b_\nu(z, w)|^2 \leqslant C_\delta.$$

If δ is sufficiently small, then there is $C > 0$ such that for all $z \in B_\delta(w)$,

$$V_{\nu-n/r}(B_\delta(w))|b_\nu(z, w)|^2 \geqslant (1 - C\delta),$$

where b_ν is a normalized Bergman kernel.

The following property is also very important for us (see, for example, [16], [17]):

$$\delta(w) \asymp \delta(z), \quad z \in B_D(w, r).$$

Also, we constantly use in this paper that (see [16], [17])

$$\int_{B_\delta(w)} \Delta^\alpha(\operatorname{Im} z)\,\mathrm{d}\nu(z) \asymp \Delta^{\alpha+2n/r}(\operatorname{Im} w), \quad \alpha > -1,\ w \in D.$$

At the end of this section we note that all our results in context of the unit ball (and even bounded pseudoconvex domains) can be seen in [14], [22], hence all our proofs are sketchy since the arguments are rather similar in all cases.

3. On some new sharp embedding theorems for Herz-type mixed norm spaces in tubular domains over symmetric cones

This main section of our work contains the formulations of all main results of this work and also contains the proofs of our main results in tube and bounded strongly pseudoconvex domains.

The theory of analytic spaces in tube domains was developed rapidly during last decades (see [1], [2], [5] and various references therein). Several Carleson-type sharp embedding theorems for such spaces are known today (see [1], [2] and references therein). The goal of this paper is to add to this list several new sharp assertions for these unbounded domains. We alert the reader that we extend our previous results in the bounded domain unit ball of $\mathbb{C}^n$ from [14]. And the proofs are rather similar. However, we found these general results interesting enough to put them in a separate paper. We need for all of our proofs, as previously in the unit ball case, various properties of r-lattices of D domain, which we listed in the previous section and various properties of analytic functions on Bergman balls from recent papers of Sehba and coauthors.

During past decades the theory of Bergman spaces in bounded strictly pseudoconvex domains with smooth boundary was also developed in many papers by various authors ([1], [2] and various references therein). Here we consider direct analogues of such spaces in context of unbounded domains. For the Bergman space theory in the unit disk, unit polydisk and in the unit ball we refer the reader to [10], [25]. One of the goals of this paper is to extend some recent results of standard weighted Bergman spaces in the tube domains in $\mathbb{C}^n$ to the case of more general $A(p,q,\alpha)$ classes Bergman-type classes in unbounded tubular domains. Note that using properties of r-lattice from [3], [5] we can get the following estimates for tube domain D:

$$\begin{aligned}(3.1)\qquad \|f\|^p_{A^p_{\alpha_1}} = \int_D |f(z)|^p\delta^\alpha(z)\,\mathrm{d}\nu(z) &\asymp \sum_{k=1}^\infty \max_{z\in B_D(a_k,r)} |f(z)|^p\delta_{\alpha_1}(B_D(a_k,r))\\ &\asymp \sum_{k=1}^\infty \int_{B_D(a_k,R)} |f(z)|^p\delta^\alpha(z)\,\mathrm{d}\nu(z),\end{aligned}$$

$0<p<\infty$, $\alpha>-1$, $\alpha_1=\alpha+nr^{-1}$, $R=c(r)$, where $c(r)$ is a certain function of r, $r\in(0,1)$.

Motivated by (3.1) we introduce a new space as follows.

Definition 3.1. Let μ be a positive Borel measure in D, $0<p,q<\infty$, $s>-1$. Fix $r\in(0;\infty)$ and an r-lattice $\{a_k\}_{k=1}^\infty$. The analytic space $A(p,q,\mathrm{d}\mu)$ is the space of all holomorphic functions f such that

$$\|f\|^q_{A(p,q,\mathrm{d}\mu)} = \sum_{k=1}^\infty \biggl(\int_{B(a_k,r)} |f(z)|^p\,\mathrm{d}\mu(z)\biggr)^{q/p} < \infty.$$

If $\mathrm{d}\mu=\delta^s(z)\,\mathrm{d}\nu(z)$, then we will denote by $A(p,q,s)$ the space $A(p,q,\mathrm{d}\mu)$. This is a Banach space for $\min(p,q)\geqslant 1$. It is clear that $A(p,p,s)=A^p_{\tilde{s}}$, $\tilde{s}=s+nr^{-1}$.

Remark 3.1. It is clear now from the discussion above and the definition of $A(p,p,s)$ spaces that these classes are independent of $\{a_k\}$ and r. But in the general case of $A(p,q,s)$ spaces the answer is unknown. For simplicity we denote $\|f\|_{A(p,q,s,a_k,r)}$ by $\|f\|_{A(p,q,s)}$.

We also have the following estimates using r-lattice:

$$\|f\|^q_{A(p,q,s)} = \sum_{k=1}^{\infty}\bigg(\int_D \chi_{B_D(a_k,r)}(z)|f(z)|^p\delta^s(z)\,\mathrm{d}\nu(z)\bigg)^{q/p}$$
$$\leqslant C\bigg(\int_D |f(z)|^p\delta^s(z)\,\mathrm{d}\nu(z)\bigg)^{q/p} = C\|f\|^q_{A^p_{\tilde{s}}}, \quad q \geqslant p,\ s > -1,$$

where $\tilde{s} = s + nr^{-1}$.

So finally we have

$$\|f\|_{A(p,q,s)} \leqslant C\|f\|_{A^p_{\tilde{s}}}, \quad q \geqslant p,\ s > -1.$$

Motivated by this estimate we pose the following very natural and more general problem (as in the case of the unit ball).

Problem: Let μ be a positive Borel measure in tube D and let $\{a_k\}_{k\in\mathbb{N}}$ be a sequence such that $B_D(a_k,r)$ is an r-lattice for tubular domain D in $\mathbb{C}^n$. Let X be a quasinormed subspace of $H(D)$ and $p,q \in (0,\infty)$. Describe all positive Borel measures such that

$$\|f\|_{A(p,q,\mathrm{d}\mu)} \leqslant C\|f\|_X. \tag{3.2}$$

For $p = q$ case see [6], [16], [17]. Some modifications of (3.2) are also interesting.

We note again that for all proofs of the assertions below we will need the properties of r-lattice, which we listed in previous sections, and various properties of Bergman balls from recent papers [16] and [17], which we also listed above. The estimate from below of Bergman kernel on Bergman balls is crucial everywhere below and as in the case of simpler domains it can be seen in [16] and [17].

All theorems of this section in very particular case of the unit ball can be seen in [14]. Moreover, the arguments of the proofs are rather similar, so we omit some proofs.

The following result (a multifunctional sharp embedding theorem for tubular domains over symmetric cones) is as far as we know new even if the amount of functions is equal to one.

Theorem 3.1. *Let μ be a positive Borel measure on D and $\{a_k\}$ be a Bergman sampling sequence forming an r-lattice. Let $\alpha > -1$, $f_i \in H(D)$, $0 < p_i, q_i < \infty$, $i = 1,\dots,m$ so that $\sum_{i=1}^{m} q_i^{-1} = 1$. Then*

$$\int_D \prod_{i=1}^{m} |f_i(z)|^{p_i}\,\mathrm{d}\mu(z) \leqslant C\prod_{i=1}^{m}\bigg[\sum_{k=1}^{\infty}\bigg(\int_{B(a_k,r)} |f_i(z)|^{p_i}\delta^{\alpha}(z)\,\mathrm{d}\nu(z)\bigg)^{q_i}\bigg]^{1/q_i}$$

if and only if

$$\mu(B_D(a_k,r)) \leqslant C\delta^{m(\alpha+2n/r)}(a_k) \tag{3.3}$$

for every $k=1,2,3,\ldots$, $r>0$.

Remark 3.2. The assertion of Theorem 3.1 can be found in paper [14] for the case of the unit ball in $\mathbb{C}^n$. For $q_i=1$, $p_i=p$, $m=1$ it can be seen in [25] for the unit ball.

Theorem 3.2. *Let* $0<q$, $s<\infty$, $q\geqslant s$, $\alpha>-1$. *Let* $\{a_k\}_{k=1}^{\infty}$ *be a sequence forming an* r*-lattice in* D. *Let* μ *be a positive Borel measure in* D. *Then*

$$\int_D |f(z)|^q \,\mathrm{d}\mu(z) \leqslant C\int_D \biggl(\int_{B_D(z,r)} |f(w)|^s \,\mathrm{d}\nu_\alpha(w)\biggr)^{q/s} \mathrm{d}\nu(z)$$

if and only if

$$\mu(B_D(a_k,r)) \leqslant C(\delta(a_k))^{q((\alpha+2n/r)/s+2n/(rq))} \tag{3.4}$$

for some constant C, $C>0$, $k\in\mathbb{N}$.

Theorem 3.3. *Let* $0<q,p<\infty$, $0<s\leqslant p<\infty$, $\beta>-1$, $\tilde\beta=\beta+nr^{-1}$. *Let* μ *be a positive Borel measure on* D. *Then we have the assertion*

$$\|f\|_{A(q,p,\mathrm{d}\mu)} \leqslant C\|f\|_{A^s_{\tilde\beta}}$$

if and only if

$$\mu(B_D(a_k,r)) \leqslant C(\delta(a_k))^{q(2n/r+\beta)/s}. \tag{3.5}$$

Remark 3.3. The estimates of these types (3.5) for a unit ball can be found in paper [14] for simpler case $p=q$, $p=s$; similar assertions are proved in [25] for the unit ball.

Remark 3.4. Our one functional theorems can be easily extended to weighted spaces with quasinorms

$$\int_D \biggl(\int_{B_D(z,r)} |f(w)|^s \,\mathrm{d}\nu_\alpha(w)\biggr)^{q/s} \mathrm{d}\nu_\beta(z), \quad \sum_{k=1}^{\infty}\biggl(\int_{B(a_k,r)} |f(z)|^p \,\mathrm{d}\mu(z)\biggr)^{q/p} \delta^\beta(a_k)$$

with some restrictions on the parameters involved. Similarly, a bit more general form can be given for our multifunctional result.

As it was mentioned above we intend to give in this paper “unbounded” versions of our earlier results proved before in the case of the unit ball in $\mathbb{C}^n$ and in bounded strictly pseudoconvex domains with smooth boundary. For this purpose we heavily use the new vital technique which was developed in very recent vital papers [16] and [17], where the so-called r-lattice was applied for the embeddings of similar type in tube domains. We note that all proofs of our theorems will not be given in this paper because of certain real similarities in arguments we used in the case of the unit ball before and here below. Note also again here as before that in the case of a unit ball all our proofs are heavily based on nice properties of r-lattice, which we listed in previous sections, but for the case of a tube.

The proofs are rather sketchy for reasons which were already indicated (see also [14] for the unit ball case).

Proof of Theorem 3.1. First suppose that (3.3) holds. Then using properties of r-lattices, which we listed in previous sections, and Bergman balls we have

$$\begin{aligned} J &= \int_D \prod_{i=1}^m |f_i(z)|^{p_i} \, \mathrm{d}\mu(z) = \sum_{k=1}^\infty \int_{B_D(a_k,r)} \prod_{i=1}^m |f_i(z)|^{p_i} \, \mathrm{d}\mu(z) \\ &\leqslant \widetilde{C} \sum_{k=1}^\infty \sup_{B_D(a_k,r)} \prod_{i=1}^m |f_i(z)|^{p_i} \mu(B_D(a_k,r)) \\ &\leqslant \widetilde{C}_1 \sum_{k=1}^\infty \mu(B_D(a_k,r)) \prod_{i=1}^m \sup_{B_D(a_k,r)} |f_i(z)|. \end{aligned}$$

Hence, using Lemma 2.3 and the comments after it and since $\delta(a_k) \asymp \delta(w)$ if $w \in B_D(a_k,R)$, we have

$$J \leqslant C \sum_{k=1}^\infty (\mu(B_D(a_k,r))) \prod_{i=1}^m \sup_{z\in B_D(a_k,r)} |f_i(z)|^{p_i};$$

$$J \leqslant \widetilde{C} \sum_{k=1}^\infty \frac{\mu(B_D(a_k,r))}{\delta^{m(\alpha+2n/r)}(a_k)} \delta^{m(\alpha+2n/r)}(a_k) \prod_{i=1}^m \int_{B_D(a_k,R)} |f_i(w)|^{p_i} \frac{\mathrm{d}\nu(w)}{(\nu(B_D(a_k,r)))^m},$$

since $(\nu(B_D(a_k,r)))^m \asymp \delta^{m2n/r}(a_k)$, $k=1,2,\ldots$,

$$\begin{aligned} J &\leqslant C_1 \sum_{k=1}^\infty \frac{\mu(B_D(a_k,r))}{\delta^{m(\alpha+2n/r)}(a_k)} \prod_{i=1}^m \int_{B_D(a_k,R)} |f_i(w)|^{p_i} \delta^\alpha(w) \, \mathrm{d}\nu(w) \\ &\leqslant C_2 \sum_{k=1}^\infty \prod_{i=1}^m \int_{B_D(a_k,R)} |f_i(w)|^{p_i} \delta^\alpha(w) \, \mathrm{d}\nu(w). \end{aligned}$$

Using the condition $\sum\limits_{i=1}^m q_i^{-1} = 1$ and Hölder's inequality for m functions we get what we need. The converse follows from the chain of equalities and estimates based again on the properties of r-lattice, which we listed in the previous section. Indeed, we have as above for f_i the test function

$$f_i(z) = (\delta^{(\alpha+2n/r)/p_i}(a_k))(B_{2(\alpha+2n/r)/p_i-n/r}(z,a_k)), \quad i = 1,2,\dots,m.$$

By the properties of r-lattice, which we listed in previous section, using Corollary 2.1 we have

$$\begin{aligned}\int_D \prod_{i=1}^m |f_i(z)|^{p_i}\,\mathrm{d}\mu(z) &\geqslant \int_{B_D(a_k,r)} (\delta^{m(\alpha+2n/r)}(a_k))(B_\tau(a_k,a_k))\,\mathrm{d}\mu(z)\\ &\geqslant \frac{\mu(B_D(a_k,r))}{\delta^{m(\alpha+2n/r)}(a_k)}, \quad \tau = 2m\Big(\alpha+\frac{2n}{r}\Big)-\frac{n}{r}.\end{aligned}$$

Hence, we get what we need. Indeed, we have the following estimates:

$$\begin{aligned}&\prod_{i=1}^m\Big(\sum_{k=1}^\infty\Big(\int_{B_D(a_k,r)}|f_i(z)|^{p_i}\delta^\alpha(z)\,\mathrm{d}\nu(z)\Big)^{q_i}\Big)^{1/q_i}\\ &\qquad\leqslant \prod_{i=1}^m\sum_{k=1}^\infty\int_{B_D(a_k,r)}|f_i(z)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z).\end{aligned}$$

Note first of all that f_i depends on k by definition. So since each $y\in D$ belongs to at most N balls $B_D(a_k,r)$, we have

$$\begin{aligned}\prod_{i=1}^m\sum_{k=1}^\infty\int_{B_D(a_k,r)}|f_i(z)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z) &\leqslant \prod_{i=1}^m\sum_{k=1}^\infty\int_D \chi_{B_D(a_k,r)}(z)|f_i(z)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z)\\ &\leqslant \prod_{i=1}^m\sum_{k=1}^N\int_D|f_i(z)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z)\end{aligned}$$

and using the definition of f_i we get

$$\begin{aligned}&\prod_{i=1}^m\sum_{k=1}^N\int_D|f_i(z)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z)\\ &\qquad\leqslant \prod_{i=1}^m\sum_{k=1}^N\delta^{\alpha+2n/r}(a_k)\int_D|B_{2(\alpha+2n/r)/p_i-n/r}(z,a_k)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z)\\ &\qquad\leqslant \prod_{i=1}^m\sum_{k=1}^N\delta^{\alpha+2n/r}(a_k)\delta^{-2n/r-\alpha}(a_k)\end{aligned}$$

since

$$\int_D|B_{2(\alpha+2n/r)/p_i-n/r}(z,a_k)|^{p_i}(\delta^\alpha(z))\,\mathrm{d}\nu(z) = c\delta^{-2n/r-\alpha}(a_k)$$

whenever $\alpha>-1$. The proof of the theorem is complete. $\square$

P r o o f of Theorem 3.2. Let (3.4) hold. We have for the same $\{a_k\}$ sequence and using the properties of r-lattice, which we listed in previous sections (Lemmas 2.2–2.4 and the remarks after Corollary 2.1),

$$\int_D |f(w)|^q \, \mathrm{d}\mu(w) \leqslant \sum_{k=1}^{\infty} \sup_{w \in B_D(a_k,r)} |f(w)|^q \mu(B_D(a_k,r)).$$

Since $qs^{-1} \geqslant 1$,

$$\begin{aligned} (3.6) \quad \sum_{k=1}^{\infty} & \sup_{w \in B_D(a_k,r)} |f(w)|^q \mu(B_D(a_k,r)) \\ & \leqslant \Big[\sum_{k=1}^{\infty} \sup_{w \in B_D(a_k,r)} |f(w)|^s (\mu(B_D(a_k,r)))^{s/q} \Big]^{q/s} \\ & \leqslant C \Big[\sum_{k=1}^{\infty} \Big(\sup_{w \in B_D(a_k,r)} |f(w)|^s \Big) \delta^{s((\alpha+2n/r)/s+2n/(rq))}(a_k) \Big]^{q/s}. \end{aligned}$$

Since $w \in D$ and D is open, there exists $\delta > 0$ such that $B_D(w, 2\tilde{\delta}) \in D$. From the mean value formula we derive (see also Lemma 2.3 and the remark after it)

$$|f(w)|^s \leqslant \frac{1}{\nu(B_D(w,\tilde{\delta}))} \int_{B_D(w,\tilde{\delta})} |f(\widetilde{w})|^s \, \mathrm{d}\nu(\widetilde{w}) \simeq \delta^{-2n/r}(w) \int_{B_D(w,\tilde{\delta})} |f(\widetilde{w})|^s \, \mathrm{d}\nu(\widetilde{w}).$$

This leads to

$$(3.7) \qquad \sup_{w \in B_D(a_k,r)} |f(w)|^s \lesssim \delta^{-2n/r}(a_k) \int_{B_D(a_k,\tilde{\delta}+r)} |f(\widetilde{w})|^s \, \mathrm{d}\nu(\widetilde{w})$$

since $\delta(w) \sim \delta(a_k)$ and $B_D(w,\delta) \subset B_D(a_k, \delta + r)$ whenever $w \in B_D(a_k, r)$. It follows from (3.6) and (3.7) that

$$\begin{aligned} \sum_{k=1}^{\infty} & \sup_{w \in B_D(a_k,r)} |f(w)|^q \mu(B_D(a_k,r)) \\ & \leqslant C \Big[\sum_{k=1}^{\infty} \delta^{s((\alpha+2n/r)/s+2n/(rq))-2n/r}(a_k) \int_{B_D(a_k,\delta+r)} |f(\widetilde{w})|^s \, \mathrm{d}\nu(\widetilde{w}) \Big]^{q/s}. \end{aligned}$$

Then we have $\delta(w) \asymp \delta(z)$, $z \in B_D(w,r)$ (see [16], [17]) and hence

$$\begin{aligned} \int_{B_D(a_k,R)} & |f(z)|^s \, \mathrm{d}\nu(z) \\ & \leqslant C \int_{B_D(a_k,R)} \Big(\int_{B_D(z,r)} |f(\widetilde{w})|^s \, \mathrm{d}\nu_\alpha(\widetilde{w}) \Big) \frac{\mathrm{d}\nu(z)}{\delta^{(\alpha+2n/r)}(z)}, \quad R = \delta + r. \end{aligned}$$

By Hölder's inequality for this R we have, using the properties of r-lattice (Lemmas 2.2–2.4 and the remarks after Corollary 2.1),

$$\left(\int_{B_D(a_k,R)}\int_{B_D(z,r)}|f(\widetilde{w})|^s\,\mathrm{d}\nu_\alpha(\widetilde{w})\frac{\mathrm{d}\nu(z)}{\delta^{2n/r}(z)}\right)^{q/s}$$
$$\leqslant \tilde{c}\int_{B_D(a_k,R)}\left(\int_{B_D(z,r)}|f(\widetilde{w})|^s\,\mathrm{d}\nu_\alpha(\widetilde{w})\right)^{q/s}(\delta^{-2n/r}(a_k))\,\mathrm{d}\nu(z).$$

Taking in previous estimate $R=\delta+r$, we shall finally obtain

$$\int_D|f(w)|^q\,\mathrm{d}\mu(w)\leqslant C\int_D\left(\int_{B_D(z,r)}|f(\widetilde{w})|^s\,\mathrm{d}\nu_\alpha(\widetilde{w})\right)^{q/s}\mathrm{d}\nu(z).$$

We show the converse. We have $\{a_k\}$, $z\in D$, $k=1,2,\dots$ and β which is big enough. Let

$$f_k(z)=\delta^{(\beta-(\alpha+2n/r))/s-2n/(rq)}(a_k)[B_{n/r}(z,a_k)]^{\widetilde{\beta}},\quad \widetilde{\beta}=\frac{\beta}{2nr^{-1}}.$$

We can choose β so that $\widetilde{\beta}$ can be integer.

Then by estimate (2.3), Lemmas 2.2–2.4 and the remarks after Corollary 2.1, we have

$$\int_D\left(\int_{B_D(w,r)}|f_k(z)|^s\,\mathrm{d}\nu_\alpha(z)\right)^{q/s}\mathrm{d}\nu(w)\leqslant C(\delta^\tau(a_k))\frac{1}{\delta^\tau(a_k)}\leqslant \text{const.},$$
$$\tau=\beta q-q\frac{\alpha+2nr^{-1}}{s}-\frac{2n}{r}.$$

Note the fact that

$$\int_{B_D(\widetilde{w},r)}\frac{\delta^s(w)\,\mathrm{d}\nu(w)}{|\Delta^\alpha((z-\overline{w})\mathrm{i}^{-1})|}\leqslant\frac{c}{|\Delta^{\widetilde{\alpha}}((z-\overline{\widetilde{w}})\mathrm{i}^{-1})|},\quad \widetilde{\alpha}=\alpha-s-\frac{2n}{r},$$

$\alpha>s+2nr^{-1}$, $z,\widetilde{w}\in D$ can be seen in [20]. Then using Corollary 2.1 we have

$$\int_D|f_k(z)|^q\,\mathrm{d}\mu(z)\geqslant C(\mu(B_D(a_k,r)))\delta^{-q((\alpha+2n/r)/s+2n/(rq))}(a_k).$$

The rest is clear (see also [14]). The proof of the theorem is complete. □

Remark 3.5. Note that in the proofs we repeat the arguments from [14] provided there in the case of much simpler domain in $\mathbb{C}^n$, namely in the unit ball in $\mathbb{C}^n$.

Proof of Theorem 3.3. Suppose (3.5) holds. Then using the properties of r-lattice, which we listed in previous sections (Lemmas 2.2–2.4 and the remarks after Corollary 2.1) we have

$$\begin{aligned}
&\left(\sum_{k=1}^{\infty}\left[\int_{B_D(a_k,r)}|f(z)|^q\,\mathrm{d}\mu(z)\right]^{p/q}\right)^{s/p}\\
&\qquad\leqslant C_1\left(\sum_{k=1}^{\infty}\max_{z\in B_D(a_k,r)}|f(z)|^p\delta^{p(2n/r+\beta)/s}(a_k)\right)^{s/p}\\
&\qquad\leqslant C_2\sum_{k=1}^{\infty}\max_{z\in B_D(a_k,r)}|f(z)|^s\delta^{(2n/r+\beta)}(a_k)\\
&\qquad\leqslant C_3\int_D|f(z)|^s\delta^{\beta}(z)\,\mathrm{d}\nu(z)\leqslant C_4\|f\|^s_{A^s_{\tilde{\beta}}(D)},\quad \beta>0,\ 0<s<\infty.
\end{aligned}$$

Conversely, using an appropriate test function $f_k(z)$ and the estimates from below of Bergman-type kernel B_s (see Corollary 2.1 and remarks therein) and using also the properties of r-lattices, which we listed in the previous section, for a test function

$$f_k(z)=\delta^{(\beta+2n/r)/s}(a_k)B_{t/s-n/r}(z,a_k),\quad z\in D,\ k=1,2,\ldots,\ t=2\Big(\beta+\frac{2n}{r}\Big),$$

where β is large enough positive number, and noting that

$$\begin{aligned}
(3.8)\qquad &\left(\int_{B_D(a_k,r)}|f(z)|^q\,\mathrm{d}\mu(z)\right)^{1/q}=C_k(f)\leqslant\left(\sum_{k=1}^{\infty}(C_k(f))^p\right)^{1/p}\\
&=C\left[\sum_{k=1}^{\infty}\left(\int_{B_D(a_k,r)}|f(z)|^q\,\mathrm{d}\mu(z)\right)^{p/q}\right]^{1/p}\leqslant c\|f\|_{A^s_{\tilde{\beta}}}.
\end{aligned}$$

Using Corollary 2.1 we get what we need.

Indeed, putting f_k into (3.8) and using the fact that $\sup\limits_k\|f_k\|_{A^s_{\tilde{\beta}}}\leqslant$ const., which follows from estimate (2.3) (see also [16] and [17]), we will get what we need. The proof of the theorem is complete. □

The careful analysis of the proofs of these embeddings in Herz-type analytic function spaces we provided above shows various similarities to our previous work of similar type in the unit ball and bounded strictly pseudoconvex domains. Nevertheless, tubular domains over symmetric cones are general unbounded as domains and our results can be seen only as direct analogues of our previous embedding theorems in bounded domains in higher dimension. Some similar results in Herz-type analytic function spaces with similar proofs are valid also in bounded symmetric domains and minimal homogeneous domains (see [11], [23], [24] for some machinery which is needed for such results and proofs).

Note finally that by similar methods some sharp reverse embeddings of the following type can be obtained:

$$\int_D \left(\int_{B_D(z,r)} |f(w)|^s \, \mathrm{d}\mu(w) \right)^{q/s} \mathrm{d}\nu(z) \leqslant C \|f\|_{A^p_\beta}$$

for some values of q, s, p, β, or

$$\left(\int_D |f(z)|^v d\mu(z) \right)^{1/v} \leqslant C_1 \sum_{k=1}^{\infty} \left(\int_{B(a_k,r)} |f(z)|^p \, \mathrm{d}\nu_\alpha(z) \right)^{q/p}$$

for some values of parameters v, p, q, α and a fixed positive Borel measure μ on T_Ω. We omit the details.

Some results of this paper can be extended by similar methods to appropriately defined analytic Herz-type function spaces on product domains $\mathbb{C}^n \times \ldots \times \mathbb{C}^n = \mathbb{C}^{mn}$.

References

[1] *A. Abate, J. Raissy, A. Saracco*: Toeplitz operators and Carleson measures in strongly pseudoconvex domains. J. Funct. Anal. *263* (2012), 3449–3491. zbl MR doi

[2] *M. Abate, A. Saracco*: Carleson measures and uniformly discrete sequences in strongly pseudoconvex domains. J. Lond. Math. Soc., II. Ser. *83* (2011), 587–605. zbl MR doi

[3] *D. Bekolle, A. Bonami, G. Garrigos, C. Nana, M. Peloso, F. Ricci*: Lecture notes on Bergman projectors in tube domains over cones: an analytic and geometric viewpoint. IMHOTEP J. Afr. Math. Pures Appl. *5* (2004), 75 pages. zbl MR

[4] *D. Bekolle, A. Bonami, G. Garrigos, F. Ricci*: Littlewood-Paley decompositions related to symmetric cones and Bergman projections in tube domains. Proc. Lond. Math. Soc., III. Ser. *89* (2004), 317–360. zbl MR doi

[5] *D. Bekolle, A. Bonami, G. Garrigos, F. Ricci, B. Sehba*: Analytic Besov spaces and Hardy-type inequalities in tube domains over symmetric cones. J. Reine Angew. Math. *647* (2010), 25–56. zbl MR doi

[6] *D. Bekolle, B. Sehba, E. Tchoundja*: The Duren-Carleson theorem in tube domains over symmetric cones. Integral Equations Oper. Theory *86* (2016), 475–494. zbl MR doi

[7] *L. Carleson*: Interpolations by bounded analytic functions and the corona problem. Ann. Math. *76* (1962), 547–559. zbl MR doi

[8] *J. A. Cima, P. R. Mercer*: Composition operators between Bergman spaces on convex domains in $\mathbb{C}^n$. J. Oper. Theory *33* (1995), 363–369. zbl MR

[9] *J. A. Cima, W. R. Wogen*: A Carleson measure theorem for the Bergman space on the ball. J. Oper. Theory *7* (1982), 157–165. zbl MR

[10] *P. L. Duren*: Extension of a theorem of Carleson. Bull. Am. Math. Soc. *75* (1969), 143–146. zbl MR doi

[11] *L. G. Gheorghe*: Interpolation of Besov spaces and applications. Matematiche *55* (2000), 29–42. zbl MR

[12] *W. W. Hastings*: A Carleson measure theorem for Bergman spaces. Proc. Am. Math. Soc. *52* (1975), 237–241. zbl MR doi

[13] *H. T. Kaptanoğlu*: Carleson measures for Besov spaces on the ball with applications. J. Funct. Anal. *250* (2007), 483–520. zbl MR doi
[14] *S. Li, R. Shamoyan*: On some estimates and Carleson type measure for multifunctional holomorphic spaces in the unit ball. Bull. Sci. Math. *134* (2010), 144–154. zbl MR doi
[15] *D. Luecking*: A technique for characterizing Carleson measures on Bergman spaces. Proc. Am. Math. Soc. *87* (1983), 656–660. zbl MR doi
[16] *C. Nana, B. F. Sehba*: Carleson embeddings and two operators on Bergman spaces of tube domains over symmetric cones. Integral Equations Oper. Theory *83* (2015), 151–178. zbl MR doi
[17] *C. Nana, B. Sehba*: Toeplitz and Hankel operators from Bergman to analytic Besov spaces of tube domains over symmetric cones. Avaible at `https://arxiv.org/abs/1508.05819`.
[18] *V. L. Oleĭnik*: Embedding theorems for weighted classes of harmonic and analytic functions. J. Sov. Math. *9* (1978), 228–243. zbl doi
[19] *V. L. Oleĭnik, B. S. Pavlov*: Embedding theorems for weighted classes of harmonic and analytic functions. J. Sov. Math. *2* (1974), 135–142. zbl doi
[20] *B. F. Sehba*: Hankel operators on Bergman spaces of tube domains over symmetric cones. Integral Equations Oper. Theory *62* (2008), 233–245. zbl MR doi
[21] *B. F. Sehba*: Bergman-type operators in tubular domains over symmetric cones. Proc. Edinb. Math. Soc., II. Ser. *52* (2009), 529–544. zbl MR doi
[22] *R. F. Shamoyan, O. R. Mihić*: On some new sharp embedding theorems in minimal and pseudoconvex domains. Czech. Math. J. *66* (2016), 527–546. zbl MR doi
[23] *S. Yamaji*: Composition operators on the Bergman spaces of a minimal bounded homogeneous domain. Hiroshima Math. J. *43* (2013), 107–127. zbl MR
[24] *S. Yamaji*: Positive Toeplitz operators on weighted Bergman spaces of a minimal bounded homogeneous domain. J. Math. Soc. Japan *64* (2013), 1101–1115. zbl MR doi
[25] *K. Zhu*: Spaces of Holomorphic Functions in the Unit Ball. Graduate Texts in Mathematics 226, Springer, New York, 2005. zbl MR doi

Authors' addresses: Romi F. Shamoyan, Department of Mathematics, Bryansk State Technical University, Bryansk 241050, Russia, e-mail: `rshamoyan@gmail.com`; Olivera Mihić, University of Belgrade, Faculty of Organizational Sciences, Jove Ilića 154, 11000 Belgrade, Serbia e-mail: `oliveradj@fon.rs`.