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A CHARACTERIZATION OF A CERTAIN REAL HYPERSURFACE OF TYPE (A_2) IN A COMPLEX PROJECTIVE SPACE

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Dedicated to Professor U-Hang Ki on the occasion of his 80th birthday

Abstract. In the class of real hypersurfaces M^{2n-1} isometrically immersed into a nonflat complex space form $\widetilde{M}_n(c)$ of constant holomorphic sectional curvature c $(\neq 0)$ which is either a complex projective space $\mathbb{C}P^n(c)$ or a complex hyperbolic space $\mathbb{C}H^n(c)$ according as $c > 0$ or $c < 0$, there are two typical examples. One is the class of all real hypersurfaces of type (A) and the other is the class of all ruled real hypersurfaces. Note that the former example are Hopf manifolds and the latter are non-Hopf manifolds. In this paper, inspired by a simple characterization of all ruled real hypersurfaces in $\widetilde{M}_n(c)$, we consider a certain real hypersurface of type (A_2) in $\mathbb{C}P^n(c)$ and give a geometric characterization of this Hopf manifold.

Keywords: ruled real hypersurface; nonflat complex space form; real hypersurfaces of type (A_2) in a complex projective space; geodesics; structure torsion; Hopf manifold

MSC 2010: 53B25, 53C40

1. Introduction

We consider a real hypersurface M^{2n-1} (with Riemannian metric g) in a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$, through an isometric immersion. We first recall the definition of ruled real hypersurfaces M^{2n-1} in $\widetilde{M}_n(c)$. A real hypersurface M is *ruled* if the holomorphic distribution $T^0M = \{X \in TM\colon\ g(X,\xi) = 0\}$ is integrable and all of its leaves (i.e., maximal integral manifolds) are locally congruent to totally geodesic complex hypersurfaces $\widetilde{M}_{n-1}(c)$ in the ambient space $\widetilde{M}_n(c)$, where ξ is

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the characteristic vector field with respect to the almost contact metric structure (φ, ξ, η, g) on M induced from the Kähler structure J of $\widetilde{M}_n(c)$.

We here recall the construction of ruled real hypersurfaces in $\widetilde{M}_n(c)$. For a real smooth curve $\gamma = \gamma(s)$, $s \in I$, parametrized by its arclength s, where I is an open interval on $\mathbb{R}$, we take the totally geodesic complex hypersurface $\widetilde{M}^{(s)}_{n-1}$ through the point $\gamma(s)$ in $\widetilde{M}_n(c)$ in such a way that the complex line spanned by $\dot{\gamma}(s)$ is perpendicular to the tangent space $T_{\gamma(s)}\widetilde{M}^{(s)}_{n-1}$. Then we get a ruled real hypersurface $M = \bigcup_{s\in \mathrm{I}} \widetilde{M}^{(s)}_{n-1}$. Of course M has singular points, that is, M is not smooth at those points. So, in general we omit such points and consider locally ruled real hypersurfaces. Adachi, the third author and Kim gave the following characterization of all ruled hypersurfaces in a nonflat complex space form (see Proposition 2 and Lemma 4 in [8]).

Proposition A. *A real hypersurface M is ruled in a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$, if and only if there exist such orthonormal vectors $v_1, v_2, \ldots, v_{2n-2}$ orthogonal to the characteristic vector ξ_p at each point p of M that the following two conditions hold:*

(1) *Every geodesic $\gamma_i = \gamma_i(s)$ on M with $p = \gamma_i(0)$ and $\dot{\gamma}_i(0) = v_i$, $1 \leqslant i \leqslant 2n-2$, is also mapped to a geodesic in the ambient space $\widetilde{M}_n(c)$.*

(2) *Every geodesic $\gamma_{ij} = \gamma_{ij}(s)$ on M with $p = \gamma_{ij}(0)$ and $\dot{\gamma}_{ij}(0) = (v_i + v_j)/\sqrt{2}$, $1 \leqslant i < j \leqslant 2n-2$, is also mapped to a geodesic in $\widetilde{M}_n(c)$.*

Motivated by Proposition A, we pose the following problem:

Problem. If we delete Condition (2) in Proposition A, is M ruled in $\widetilde{M}_n(c)$?

In this paper, we give a negative answer to this problem in the case of $c > 0$. We present a counterexample M^{2n-1} to Problem, which is locally congruent to a tube of radius $\pi/(2\sqrt{c})$ around a totally geodesic $\mathbb{C}P^{(n-1)/2}(c)$ in $\mathbb{C}P^n(c)$, where n $(\geqslant 3)$ is odd. This real hypersurface is a member of real hypersurfaces of type (A_2) in $\mathbb{C}P^n(c)$.

The main purpose of this paper is to characterize this hypersurface with Condition (1) in Proposition A (for details, see Theorem). Note that we have not been able to solve the above problem until now when $c < 0$.

2. Terminologies and fundamental results on real hypersurfaces

Let M^{2n-1} be a real hypersurface with unit normal local vector field $\mathcal{N}$ of a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$. The Riemannian connections $\widetilde{\nabla}$ of $\widetilde{M}_n(c)$ and ∇ of M are related by

$$\widetilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)\mathcal{N} \tag{2.1}$$

and

$$\widetilde{\nabla}_X \mathcal{N} = -AX \tag{2.2}$$

for all vector fields X and Y on M, where g denotes the metric induced from the standard Riemannian metric of $\widetilde{M}_n(c)$ and A is the shape operator of M in $\widetilde{M}_n(c)$ associated with $\mathcal{N}$. On M an almost contact metric structure (φ, ξ, η, g) associated with $\mathcal{N}$ is canonically induced from the Kähler structure J of the ambient space $\widetilde{M}_n(c)$. It is defined by

$$g(\varphi X, Y) = g(JX, Y), \quad \xi = -J\mathcal{N} \quad \text{and} \quad \eta(X) = g(\xi, X) = g(JX, \mathcal{N}).$$

It follows from the Gauss formula (2.1), the Weingarten formula (2.2) and the property $\widetilde{\nabla} J = 0$ that

$$\nabla_X \xi = \varphi AX \tag{2.3}$$

and

$$(\nabla_X \varphi)Y = \eta(Y)AX - g(AX, Y)\xi \tag{2.4}$$

for each $X \in TM$.

We call the eigenvalues and eigenvectors of the shape operator A the *principal curvatures* and *principal curvature vectors* of M in $\widetilde{M}_n(c)$, respectively. Here and in the following, we set $V_\lambda := \{X \in TM \colon AX = \lambda X\}$. We usually call M a *Hopf hypersurface* if the characteristic vector ξ of M is a principal curvature vector at each point of M. The following are typical examples of Hopf hypersurfaces in a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$:

In $\mathbb{C}P^n(c)$,

(A_1) a geodesic sphere $G(r)$, $0 < r < \pi/\sqrt{c}$;

(A_2) a tube of constant radius r, $0 < r < \pi/\sqrt{c}$, around a totally geodesic $\mathbb{C}P^l(c)$, $1 \leqslant l \leqslant n-2$.

In $\mathbb{C}H^n(c)$,

(A_0) the horosphere HS in $\mathbb{C}H^n(c)$;
($A_{1,0}$) a geodesic sphere $G(r)$ of radius r, $0 < r < \infty$;
($A_{1,1}$) a tube of radius r, $0 < r < \infty$, around a totally geodesic $\mathbb{C}H^{n-1}(c)$;
(A_2) a tube of radius r, $0 < r < \infty$, around a totally geodesic $\mathbb{C}H^l(c)$, $1 \leqslant l \leqslant n-2$.

Unifying these examples, we call them *real hypersurfaces of type* (A) in $\widetilde{M}_n(c)$. The following theorem shows the importance of these real hypersurfaces.

Theorem A ([6], [9]). *Let M^{2n-1} be a real hypersurface in a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$. Then the length of the derivative of the shape operator A of M satisfies $\|\nabla A\|^2 \geqslant \frac{1}{4}c^2(n-1)$ at each point of M. In particular, $\|\nabla A\|^2 = c^2(n-1)/4$ holds on M if and only if M is locally congruent to a real hypersurface of type* (A).

For later use we prepare the following lemma (cf. [6], [9]):

Lemma A. *For a real hypersurface M isometrically immersed into a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$ the following three conditions are mutually equivalent:*

(1) *M is of type* (A)*;*
(2) *$\varphi A = A\varphi$, where φ is the structure tensor induced from the Kähler structure J on $\widetilde{M}_n(c)$;*
(3) *$g((\nabla_X A)Y, Z) = \frac{1}{4}c\{-\eta(Y)g(\varphi X, Z) - \eta(Z)g(\varphi X, Y)\}$ for all X, Y and $Z \in TM$.*

The real hypersurfaces of type (A) are fundamental examples of homogeneous real hypersurfaces in $\widetilde{M}_n(c)$, that is, they are orbits of some subgroups of the full isometry group $I(\widetilde{M}_n(c))$ of this ambient space (cf. [5]). The classification theorems of all homogeneous real hypersurfaces in $\widetilde{M}_n(c)$ are given [4], [10].

In order to prove our Theorem, for a geodesic γ on a real hypersurface M in $\widetilde{M}_n(c)$, we recall the notion of the *structure torsion* $\varrho_\gamma = g(\dot\gamma(s), \xi_{\gamma(s)})$. Note that the structure torsion ϱ_γ is constant along each geodesic γ on an arbitrary real hypersurface of type (A) in $\widetilde{M}_n(c)$ (see the proof of our Theorem).

At the end of this section we review the definition of circles in Riemannian geometry. Let $\gamma = \gamma(s)$ be a smooth real curve parametrized by its arclength s on a Riemannian manifold N with Riemannian metric g. If the curve γ satisfies the following ordinary differential equations with some nonnegative constant k:

$$\nabla_{\dot\gamma}\dot\gamma = kY_s \quad \text{and} \quad \nabla_{\dot\gamma}Y_s = -k\dot\gamma, \tag{2.5}$$

where $\nabla_{\dot\gamma}$ is the covariant differentiation along γ with respect to ∇ of N and Y_s is the so-called unit principal normal vector of γ, we call γ a circle of curvature k on N. We regard a geodesic as a circle of null curvature. By virtue of the existence and uniqueness of solutions to ordinary differential equations we can see that for each point $p \in N$, an arbitrary positive constant k and every pair of orthonormal vectors X and Y of T_pN, there exists locally the unique circle $\gamma = \gamma(s)$ on N satisfying the initial condition that $\gamma(0) = p$, $\dot\gamma(0) = X$ and $Y_0 = Y$.

Adachi, Udagawa and the third author studied circles in a nonflat complex space form (for details, see [1], [2]).

3. Statement of result

Our aim here is to prove the following:

Theorem. *Let M^{2n-1} be a real hypersurface isometrically immersed into $\mathbb{C}P^n(c)$ with an odd number n $(\geqslant 3)$. Then M is locally congruent to the tube of constant radius $\pi/(2\sqrt{c})$ around the totally geodesic $\mathbb{C}P^{(n-1)/2}(c)$ if and only if M satisfies the following two conditions* (1) *and* (2)*:*

(1) *M is a Hopf manifold.*

(2) *At each point $p \in M$ there exist such orthonormal vectors $v_1, v_2, \dots, v_{2n-2}$ of T_pM which are perpendicular to the characteristic vector ξ_p that they satisfy the following conditions* 2a) *and* 2b) :

  2a) *Every geodesic $\gamma_i = \gamma_i(s)$, $1 \leqslant i \leqslant 2n-2$, on M with the initial condition that $\gamma_i(0) = p$ and $\dot\gamma_i(0) = v_i$ is also mapped to a geodesic in $\mathbb{C}P^n(c)$.*

  2b) *Every geodesic $\gamma_{ij} = \gamma_{ij}(s)$ on M through $p = \gamma_{ij}(0)$ in the direction of $v_i + v_j$, $1 \leqslant i \leqslant j \leqslant 2n-2$, has constant structure torsion $\varrho_{\gamma_{ij}} := g(\dot\gamma_{ij}, \xi)$ along the curve γ_{ij}. Here, needless to say, when $i = j$ or $i < j$, $\dot\gamma_{ij}(0) = v_i$ $\dot\gamma_{ij}(0) = (v_i + v_j)/\sqrt{2}$, respectively.*

P r o o f. Before proving our Theorem we shall verify the following two properties of real hypersurfaces of type (A) in a nonflat complex space form $\widetilde{M}_n(c)$, $n \geqslant 2$:

(I) For every geodesic $\gamma = \gamma(s)$ on an arbitrary real hypersurface of type (A) the structure torsion ϱ_γ is constant along the curve γ.

(II) Evey geodesic $\gamma = \gamma(s)$ on an arbitrary real hypersurface of type (A) whose initial vector $\dot\gamma(0)$ is a principal curvature vector with principal curvature λ orthogonal to $\xi_{\gamma(0)}$, is mapped to a circle of positive curvature $|\lambda|$ in the ambient space $\widetilde{M}_n(c)$.

We first prove (I). It follows from (2.3), Lemma A, the symmetry of A and the skew symmetry of φ that

$$\begin{aligned}\dot\gamma\varrho_\gamma &= \nabla_{\dot\gamma}(g(\dot\gamma,\xi)) = g(\nabla_{\dot\gamma}\dot\gamma,\xi) + g(\dot\gamma,\nabla_{\dot\gamma}\xi) = g(\dot\gamma,\varphi A\dot\gamma)\\ &= g(\dot\gamma,A\varphi\dot\gamma) = g(A\dot\gamma,\varphi\dot\gamma) = -g(\varphi A\dot\gamma,\dot\gamma) = 0.\end{aligned}$$

Next, we shall prove (II). We set $Av=\lambda v$ with $v=\dot\gamma(0)$ for each geodesic γ stated in (II). Note that this geodesic γ on M satisfies that $g(\dot\gamma(s),\xi_\gamma(s))=0$ for each s (see (I)). This, together with Lemma A, implies

$$\begin{aligned}\dot\gamma\|A\dot\gamma-\lambda\dot\gamma\|^2 &= \nabla_{\dot\gamma}(g(A\dot\gamma-\lambda\dot\gamma,A\dot\gamma-\lambda\dot\gamma))\\ &= 2g((\nabla_{\dot\gamma}A)\dot\gamma,A\dot\gamma-\lambda\dot\gamma) = 2g((\nabla_{\dot\gamma}A)\dot\gamma,A\dot\gamma)\\ &= \frac{c}{2}\{-\eta(\dot\gamma)g(\varphi\dot\gamma,A\dot\gamma)-\eta(A\dot\gamma)g(\varphi\dot\gamma,\dot\gamma)\} = 0.\end{aligned}$$

So, from $A\dot\gamma(0)-\lambda\dot\gamma(0)=Av-\lambda v=0$ we can see that every geodesic γ stated in (II) satisfies $A\dot\gamma(s)=\lambda\dot\gamma(s)$ for each s. Then, by virtue of Gauss formula (2.1) and Weingarten formula (2.2) we can see that every geodesic γ stated in (II) is mapped to a circle of positive curvature $|\lambda|$. We remark that this λ is either $\frac12\sqrt{c}\cot(\sqrt{c}\,r/2)$ or $-\frac12\sqrt{c}\tan(\sqrt{c}\,r/2)$ $(0<r<\pi/\sqrt{c})$ in the case of $c>0$ while it is either $\frac12\sqrt{c}\coth(\sqrt{c}\,r/2)$ or $\frac12\sqrt{c}\tanh(\sqrt{c}\,r/2)$ $(0<r<\infty)$ in the case of $c<0$.

We are now in a position to prove our Theorem.

($\Rightarrow$) By assumption, our real hypersurface M has three distinct constant principal curvatures $\delta=0$, $\lambda_1=\sqrt{c}/2$ and $\lambda_2=-\sqrt{c}/2$, where $A\xi=0$ and $\dim V_{\sqrt{c}/2}=\dim V_{-\sqrt{c}/2}=n-1$. Moreover, $\varphi V_{\lambda_i}=V_{\lambda_i}$, $i=1,2$. At each fixed point p of M we take arbitrary orthonormal bases $\{e_1,e_2,\dots,e_{n-1}\}$ and $\{f_1,f_2,\dots,f_{n-1}\}$ of $V_{\sqrt{c}/2}$ and $V_{-\sqrt{c}/2}$, respectively. We here set an orthonormal basis $v_1,\dots,v_{n-1},\dots,v_{2n-2}$ of $T^0_pM:=\{X\in T_pM\colon\ X\perp\xi_p\}$ in such a way that $v_j=(e_j+f_j)/\sqrt{2}$, $v_{n-1+j}=(e_j-f_j)/\sqrt{2}$, $j=1,2,\dots,n-1$. Furthermore, we can see easily $g(Av_i,v_i)=0$ for all $i\in\{1,2,\dots,2n-2\}$. On the other hand, in view of Lemma A(3) we see that $g((\nabla_XA)X,X)=0$ for all $X\in TM$, so that for every geodesic γ on each real hypersurface M of type (A) the function $g(A\dot\gamma(s),\dot\gamma(s))$ is constant along the curve γ. Then every geodesic γ_i on our real hypersurface M with $\gamma_i(0)=p$ and $\dot\gamma_i(0)=v_i$, $i=1,2,\dots,2n-2$, is also mapped to a geodesic in the ambient space $\mathbb{C}P^n(c)$. Thus we have proved Condition 2a). The other Conditions (1) and 2b) are obvious (see the hypothesis and (I) in the above discussion).

($\Leftarrow$) We first make use of Condition 2b) in the case of $i=j$. Our computation here is due to [3], [7]. Note that $\dot\gamma_{ii}(0)=v_i$. Then, from (2.3) and Condition 2b) we obtain

$$\varrho'_{\gamma_{ii}} = \frac{\mathrm{d}}{\mathrm{d}s}(g(\dot\gamma_{ii}(s),\xi)) = g(\dot\gamma_{ii}(s),\nabla_{\dot\gamma_{ii}}\xi) = g(\dot\gamma_{ii}(s),\varphi A\dot\gamma_{ii}(s)) = 0,$$

so that at $s=0$ we have $g(v_i, \varphi A v_i) = 0$. On the other hand, we know that

$$\begin{aligned} g(v_i, \varphi A v_i) &= \tfrac{1}{2}\{g(\varphi A v_i, v_i) + g(v_i, \varphi A v_i)\} \\ &= \tfrac{1}{2} g((\varphi A - A\varphi) v_i, v_i). \end{aligned}$$

Hence we obtain

$$g((\varphi A - A\varphi) v_i, v_i) = 0, \quad 1 \leqslant i \leqslant 2n-2. \tag{3.1}$$

Applying the same discussion as above to every geodesic $\gamma_{ij} = \gamma_{ij}(s)$ with $i < j$, we find

$$g\Big((\varphi A - A\varphi)\frac{v_i + v_j}{\sqrt{2}}, \frac{v_i + v_j}{\sqrt{2}}\Big) = 0,$$

so that

$$g((\varphi A - A\varphi)(v_i + v_j), v_i + v_j) = 0 \quad \text{for } 1 \leqslant i < j \leqslant 2n-2. \tag{3.2}$$

It follows from equations (3.1), (3.2) and the fact $\varphi A - A\varphi$ is symmetric that

$$g((\varphi A - A\varphi) v_i, v_j) = 0 \quad \text{for } 1 \leqslant i < j \leqslant 2n-2. \tag{3.3}$$

Furthermore, by Condition (1) the following holds trivially:

$$(\varphi A - A\varphi)\xi = 0. \tag{3.4}$$

Then, by virtue of equations (3.1), (3.3) and (3.4) we can see that M is of type (A) (see Lemma A), namely M is locally congruent to either a real hypersurface of type (A_1) or type (A_2). We shall check Condition 2a) for these two real hypersurfaces one by one.

Let M be of type (A_1). So our real hypersurface M is locally congruent to a geodesic sphere $G(r)$ of radius r, $0 < r < \pi/\sqrt{c}$. Then it is known that $A\xi = \sqrt{c}\cot(\sqrt{c}\,r)\xi$ and $AX = \frac{1}{2}\sqrt{c}\cot(\frac{1}{2}\sqrt{c}\,r)X$ for all $X (\in TM)$ perpendicular to ξ. By (II) we find that every geodesic $\gamma = \gamma(s)$ with initial vector $\dot\gamma(0)$ orthogonal to $\xi_{\gamma(0)}$ is mapped to a circle of positive curvature $\frac{1}{2}\sqrt{c}\cot(\frac{1}{2}\sqrt{c}\,r)$ in the ambient space $\mathbb{C}P^n(c)$, which implies that our real hypersurface does not satisfy Condition 2a).

Let M be of type (A_2). So our real hypersurface is locally congruent to a tube of radius r, $0 < r < \pi/\sqrt{c}$, around a total geodesic $\mathbb{C}P^l(c)$, $1 \leqslant l \leqslant n-2$. Then it is known that the tangent bundle TM is decomposed as the orthogonal direct sum: $TM = \{\xi\}_{\mathbb{R}} \oplus V_{\lambda_1} \oplus V_{\lambda_2}$, where $A\xi = \sqrt{c}\cot(\sqrt{c}\,r)\xi$, $\lambda_1 = \frac{1}{2}\sqrt{c}\cot(\sqrt{c}\,r/2)$, $\lambda_2 = -\frac{1}{2}\sqrt{c}\tan(\sqrt{c}\,r/2)$, $\dim V_{\lambda_1} = 2n-2l-2$, $\dim V_{\lambda_2} = 2l$, $\varphi V_{\lambda_i} = V_{\lambda_i}$, $i = 1, 2$.

Again by using (II) we see that the initial vector of every geodesic $\gamma_i = \gamma_i(s)$, $i = 1, 2, \ldots, 2n-2$, in Condition 2a) must be expressed as $\dot{\gamma}_i(0) = au + bv$, where u, v are unit vectors with $u \in V_{\lambda_1}$, $v \in V_{\lambda_2}$, and without loss of generality we may suppose that a is positive. As a matter of course $a^2 + b^2 = 1$ and $b \neq 0$. By our argument and the fact that M is of type (A) we see that the above geodesic $\gamma_i = \gamma_i(s)$ on M is also mapped to a geodesic in the ambient space $\mathbb{C}P^n(c)$ if and only if $g(A\dot{\gamma}_i(0), \dot{\gamma}_i(0)) = 0$. Then by two equalities $\dot{\gamma}_i(0) = au + bv$ and $A\dot{\gamma}_i(0) = a\lambda_1 u + b\lambda_2 v$ we get $a = \sin(\sqrt{c}\,r/2)$ and $b = \pm\cos(\sqrt{c}\,r/2)$, so that the initial vector $\dot{\gamma}_i(0)$ is written in the form either $\dot{\gamma}_i(0) = \sin(\sqrt{c}\,r/2)u + \cos(\sqrt{c}\,r/2)v$ or $\dot{\gamma}_i(0) = \sin(\sqrt{c}\,r/2)u - \cos(\sqrt{c}\,r/2)v$. However, in general these two unit vectors are *not* orthogonal. We can see easily that M satisfies Condition 2a) if and only if these two vectors are orthogonal and $\dim V_{\lambda_1} = \dim V_{\lambda_2}$. Therefore we conclude that $r = \pi/(2\sqrt{c})$ and $l = (n-1)/2$. Thus we have proved our Theorem. $\square$

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