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DISJOINT HYPERCYCLIC POWERS OF WEIGHTED TRANSLATIONS ON GROUPS

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Abstract. Let G be a locally compact group and let $1 \leqslant p < \infty$. Recently, Chen et al. characterized hypercyclic, supercyclic and chaotic weighted translations on locally compact groups and their homogeneous spaces. There has been an increasing interest in studying the disjoint hypercyclicity acting on various spaces of holomorphic functions. In this note, we will study disjoint hypercyclic and disjoint supercyclic powers of weighted translation operators on the Lebesgue space $L^p(G)$ in terms of the weights. Sufficient and necessary conditions for disjoint hypercyclic and disjoint supercyclic powers of weighted translations generated by aperiodic elements on groups will be given.

Keywords: disjoint hypercyclic powers of weighted translations; aperiodic element; locally compact group

MSC 2010: 47A16, 47B38, 46E15

1. Introduction

Let T be a continuous linear self-map on a separable infinite dimensional Banach space X and let T^n denote the nth iterate of T. If there exists a vector $x \in X$ such that the orbit $\operatorname{orb}(T,x) = \{T^n x\colon\ n = 0,1,\ldots\}$ is dense in X, then T is called hypercyclic. Such a vector x is said to be hypercyclic for T. Besides, for every pair U,V of nonempty open subsets of X, if there is a nonnegative integer m such that $T^m(U) \cap V \neq \emptyset$, then we call T topologically transitive. It is well known that an operator T is hypercyclic if and only if it is topologically transitive. A stronger condition is the following: the operator T on X is called topologically mixing if for every pair of nonempty open subsets U and V of X there is $m \in \mathbb{N}$ such that $T^n(U)$

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meets V for each $n \geqslant m$. Hypercyclicity and supercyclicity have been studied by many authors; we refer to [1], [10], [18] for surveys.

Hypercyclic (respectively, supercyclic) operators $T_1, \dots, T_N$, $N \geqslant 2$, acting on the same space X are said to be disjoint or d-hypercyclic (respectively, d-supercyclic) provided there is some $x \in X$ for which the vector $(x, \dots, x) \in X^N$ is hypercyclic (respectively, supercyclic) for the direct sum operator $\bigoplus_{i=1}^{N} T_i$ acting on the product space X^N, endowed with the product topology. Besides, we say that operators $T_1, \dots, T_N$ in $B(X)$ are d-topologically transitive provided for any nonempty open subsets $V_0, \dots, V_N$ of X there exists $m \in \mathbb{N}$ such that

$$V_0 \cap T_1^{-m}(V_1) \cap \ldots \cap T_N^{-m}(V_N) \neq \emptyset.$$

If $T_1, \dots, T_N$ satisfy the stronger condition that

$$V_0 \cap T_1^{-m}(V_1) \cap \ldots \cap T_N^{-m}(V_N) \neq \emptyset$$

for some m onwards, then $T_1, \dots, T_N$ are said to be d-mixing. There has been an increasing interest in studying the disjoint hypercyclicity acting on different spaces of holomorphic functions. For example, disjoint hypercyclicity was studied in [2], [3], [4], [16], [17]. Besides, disjoint hypercyclic and supercyclic powers of weighted backward shifts were also characterized in [5], [6], [15].

Recently, hypercyclic, supercyclic and chaotic weighted translations on locally compact groups and their homogeneous spaces were characterized in [8], [9], [7]. And Liang et al. characterized d-hypercyclicity and d-supercyclicity of finite tuples of weighted translations generated by aperiodic elements in [12], [14]. Inspired by their work, we characterize disjoint hypercyclic powers of weighted translations on groups in this paper by developing further the results in [9], [7].

Throughout, let G be a locally compact group with identity e and a right-invariant Haar measure λ. Since a complex Banach space admits a hypercyclic operator if and only if it is separable and infinite-dimensional, the question of hypercyclicity is meaningful for the complex Lebesgue space $L^p(G)$, with respect to λ, only when G is second countable and $1 \leqslant p < \infty$. A bounded measurable function $w\colon G \to (0, \infty)$ is called a *weight* on G. Let $a \in G$ and let δ_a be the unit point mass at a. A weighted translation on G is a weighted convolution operator $T_{a,w}\colon L^p(G) \to L^p(G)$ defined by

$$T_{a,w}(f) := wT_a(f), \quad f \in L^p(G),$$

where w is a weight on G and $T_a(f) = f * \delta_a \in L^p(G)$ is the convolution:

$$(f * \delta_a)(x) = \int_G f(xy^{-1}) \,\mathrm{d}\delta_a(y) = f(xa^{-1}), \quad x \in G.$$

An element a in a group G is called a torsion element if it is of finite order. In a locally compact group G, an element $a \in G$ is called periodic [11] (or compact [13], 9.9) if the closed subgroup $G(a)$ generated by a is compact. We call an element in G aperiodic if it is not periodic. For discrete groups, periodic elements and torsion elements are identical; in other words, aperiodic elements are nontorsion elements. However, nontorsion elements in nondiscrete groups need not be aperiodic. It has been shown in [9], Lemma 1.1, that a weighted translation operator is not hypercyclic if it is generated by a torsion element. Our goal in this paper is to characterize disjoint hypercyclic powers of weighted translations generated by aperiodic elements on groups.

2. Disjoint hypercyclic powers of weighted translations

It has been shown in [9], Lemma 2.1, that an element a in a locally compact group G is aperiodic if and only if for any compact subset $K \subseteq G$, there exists $m \in \mathbb{N}$ such that $K \cap Ka^n = \emptyset$ (equivalently, $K \cap Ka^{-n} = \emptyset$) for $n > m$. In this section, we will make use of the equivalence of dense d-hypercyclicity and d-topological transitivity [6] to obtain the main result. We are now ready to give sufficient and necessary conditions for disjoint hypercyclic powers of weighted translations generated by aperiodic elements on groups.

Theorem 2.1. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. For each $1 \leqslant l \leqslant N$, let $w_l\colon G \to (0, \infty)$ be a weight on G and T_{a,w_l} a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) *$T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ are densely d-hypercyclic.*

(ii) *For $1 \leqslant l \leqslant N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim\limits_{k\to\infty} \lambda(E_k)$ and for the sequences*

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,n} := \biggl(\prod_{s=0}^{r_l n-1} w_l * \delta_a^s\biggr)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = \lim_{k\to\infty} \|\widetilde{\varphi}_{l,n_k}|_{E_k}\|_\infty = 0, \tag{2.1}$$

and, if $1 \leqslant s < l < N$,

$$\lim_{k\to\infty} \biggl\| \frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k-1} w_l * \delta_a^t}\biggr|_{E_k}\biggr\|_\infty = \lim_{k\to\infty} \biggl\| \frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k-1} w_s * \delta_a^t}\biggr|_{E_k}\biggr\|_\infty = 0. \tag{2.2}$$

P r o o f. (ii) $\Rightarrow$ (i). By Proposition 2.3 in [6], we show that $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ are d-topologically transitive. Let $V_0, \ldots, V_N$ be nonempty open subsets of $L^p(G)$. Since the space $C_c(G)$ of continuous functions on G with compact support is dense in $L^p(G)$, we can pick $f, g_1, \ldots, g_N \in C_c(G)$ with $f \in V_0$, $g_1 \in V_1, \ldots, g_N \in V_N$. Let K be the union of the compact supports of $f, g_1, \ldots, g_N$ and let $\chi_K \in L^p(G)$ be the characteristic function of K. For $1 \leqslant l \leqslant N$ and a compact subset K of G, let (E_k) and (n_k) be as in (2.1) and (2.2).

Due to the aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap K a^{\pm n} = \emptyset$ for all $n > M$.

For $1 \leqslant l \leqslant N$, we define a self-map S_{a,w_l} on the subspace $L_c^p(G)$ consisting of functions in $L^p(G)$ with compact support by

$$S_{a,w_l}(h) = \frac{h}{w_l} * \delta_{a^{-1}}, \quad h \in L_c^p(G)$$

so that

$$T_{a,w_l}^{r_l n_k} S_{a,w_l}^{r_l n_k}(h) = h, \quad h \in L_c^p(G).$$

We claim that (2.1) and (2.2) imply the following four equalities:

$$\begin{gathered}
\lim_{k\to\infty} \|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p = 0; \\
\lim_{k\to\infty} \|S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\|_p = 0; \\
\lim_{k\to\infty} \|T_{a,w_l}^{r_l n_k} S_{a,w_s}^{r_s n_k}(g_s\chi_{E_k})\|_p = 0; \\
\lim_{k\to\infty} \|T_{a,w_s}^{r_s n_k} S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\|_p = 0.
\end{gathered}$$

We prove the first of the four equalities here; the other ones follow similarly. Since $\lim\limits_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = 0$, given any $\varepsilon > 0$, there exists a positive integer $m \in \mathbb{N}$ such that $n_k > M$ and $\varphi_{l,n_k}^p < \varepsilon/\|f\|_p^p$ on E_k if $k > m$. Hence

$$\begin{aligned}
&\|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p^p \\
&\quad = \int_{E_k a^{r_l n_k}} |w_l(x)w_l(xa^{-1})\ldots w_l(xa^{-(r_l n_k - 1)})|^p |f(xa^{-r_l n_k})|^p \,\mathrm{d}\lambda(x) \\
&\quad = \int_{E_k} |w_l(xa^{r_l n_k})w_l(xa^{r_l n_k - 1})\ldots w_l(xa)|^p |f(x)|^p \,\mathrm{d}\lambda(x) \\
&\quad = \int_{E_k} |\varphi_{l,n_k}^p(x)||f(x)|^p \,\mathrm{d}\lambda(x) < \varepsilon, \quad \text{for } k > m.
\end{aligned}$$

The first equality follows by the arbitrariness of ε.

For each $k \in \mathbb{N}$, let

$$v_k = f\chi_{E_k} + \sum_{i=1}^{N} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) \in L^p(G).$$

Then

$$\|v_k - f\|_p^p \leqslant \|f\|_\infty^p \lambda(K \setminus E_k) + \sum_{i=1}^{N} \|S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k})\|_p^p$$

and

$$\begin{aligned}\|T_{a,w_l}^{r_l n_k} v_k - g_l\|_p^p &\leqslant \|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p^p + \left\|\sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) - g_l\right\|_p^p \\ &\leqslant \|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p^p + \|g_l\|_\infty^p \lambda(K \setminus E_k) + \sum_{i \neq l}^{N} \|T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k})\|_p^p.\end{aligned}$$

Hence $\lim\limits_{k\to\infty} v_k = f$ and $\lim\limits_{k\to\infty} T_{a,w_l}^{r_l n_k} v_k = g_l$, which implies

$$V_0 \cap T_{a,w_1}^{-r_1 n_k}(V_1) \cap \ldots \cap T_{a,w_N}^{-r_N n_k}(V_N) \neq \emptyset \quad \text{for some } k.$$

(i) $\Rightarrow$ (ii). Let $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ be densely d-hypercyclic. Let $K \subseteq G$ be a compact set with $\lambda(K) > 0$. Let $\varepsilon > 0$. By the aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap Ka^{\pm n} = \emptyset$ for all $n > M$. Let $\chi_K \in L^p(G)$ be the characteristic function of K. Choose $0 < \delta < \varepsilon/(1+\varepsilon)$. By assumption, there exists a d-hypercyclic vector $f \in L^p(G)$ and some $m > M$ such that for $1 \leqslant l \leqslant N$,

$$\|f - \chi_K\|_p < \delta^2 \quad \text{and} \quad \|T_{a,w_l}^{r_l m} f - \chi_K\|_p < \delta^2. \tag{2.3}$$

Let $A_\delta = \{x \in K\colon |f(x) - 1| \geqslant \delta\}$. Then we have

$$|f(x)| > 1 - \delta \quad (x \in K \setminus A_\delta) \tag{2.4}$$

and $\lambda(A_\delta) < \delta^p$, since

$$\begin{aligned}\delta^{2p} > \|f - \chi_K\|_p^p &= \int_G |f(x) - \chi_K(x)|^p \,\mathrm{d}\lambda(x) \\ &\geqslant \int_K |f(x) - 1|^p \,\mathrm{d}\lambda(x) \geqslant \int_{A_\delta} |f(x) - 1|^p \,\mathrm{d}\lambda(x) \geqslant \delta^p \lambda(A_\delta).\end{aligned}$$

Similarly, if $B_\delta = \{x \in G \setminus K\colon |f(x)| \geqslant \delta\}$, then we have

$$|f(x)| < \delta \quad \text{for } x \in (G \setminus K) \setminus B_\delta \tag{2.5}$$

and $\lambda(B_\delta) < \delta^p$.

Let $C_{l,m,\delta} = \{x \in K\colon |\widetilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m}) - 1| \geqslant \delta\}$. Then we have

$$\widetilde{\varphi}_{l,m}(x)^{-1}|f(xa^{-r_l m})| > 1 - \delta \quad (x \in K \setminus C_{l,m,\delta}) \tag{2.6}$$

and $\lambda(C_{l,m,\delta}) < \delta^p$. In fact,

$$\begin{aligned}
\delta^{2p} &> \|T^{r_l m}_{a,w_l} f - \chi_K\|_p^p = \int_G |T^{r_l m}_{a,w_l} f(x) - \chi_K(x)|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \int_{C_{l,m,\delta}} |w_l(x) w_l(xa^{-1}) \dots w_l(xa^{-(r_l m-1)}) f(xa^{-r_l m}) - 1|^p \,\mathrm{d}\lambda(x) \\
&= \int_{C_{l,m,\delta}} |\widetilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m}) - 1|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \delta^p \lambda(C_{l,m,\delta}).
\end{aligned}$$

Let $D_{l,m,\delta} = \{x \in K\colon |\varphi_{l,m}(x) f(x)| \geqslant \delta\}$. Then we have

$$\varphi_{l,m}(x)|f(x)| < \delta \quad (x \in K \setminus D_{l,m,\delta}) \tag{2.7}$$

and $\lambda(D_{l,m,\delta}) < \delta^p$. In fact, since $K \cap Ka^{r_l m} = \emptyset$, we deduce

$$\begin{aligned}
\delta^{2p} &> \int_G |w_l(x) w_l(xa^{-1}) \dots w_l(xa^{-(r_l m-1)}) f(xa^{-r_l m}) - \chi_K(x)|^p \,\mathrm{d}\lambda(x) \\
&= \int_G |w_l(xa^{r_l m}) w_l(xa^{r_l m-1}) \dots w_l(xa) f(x) - \chi_K(xa^{r_l m})|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \int_{D_{l,m,\delta}} |w_l(xa^{r_l m}) w_l(xa^{r_l m-1}) \dots w_l(xa) f(x)|^p \,\mathrm{d}\lambda(x) \\
&= \int_{D_{l,m,\delta}} |\varphi_{l,m}(x) f(x)|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \delta^p \lambda(D_{l,m,\delta}).
\end{aligned}$$

Let $F_{l,m,\delta} = \{x \in G \setminus K\colon |\widetilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m})| \geqslant \delta\}$. Then we have

$$|\widetilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m})| < \delta \quad \text{for } x \in (G \setminus K) \setminus F_{l,m,\delta} \tag{2.8}$$

and $\lambda(F_{l,m,\delta}) < \delta^p$, since

$$\begin{aligned}
\delta^{2p} &> \int_{G\setminus K} |w_l(x) w_l(xa^{-1}) \dots w_l(xa^{-(r_l m-1)}) f(xa^{-r_l m})|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \int_{F_{l,m,\delta}} |w_l(x) w_l(xa^{-1}) \dots w_l(xa^{-(r_l m-1)}) f(xa^{-r_l m})|^p \,\mathrm{d}\lambda(x) \\
&= \int_{F_{l,m,\delta}} |\widetilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m})|^p \,\mathrm{d}\lambda(x) \\
&\geqslant \delta^p \lambda(F_{l,m,\delta}).
\end{aligned}$$

Now, (2.5), (2.6) and the fact that $K \cap Ka^{-r_l m} = \emptyset$ imply that

$$\widetilde{\varphi}_{l,m}(x) < \frac{|f(xa^{-r_l m})|}{1-\delta} < \frac{\delta}{1-\delta} < \varepsilon \quad \text{for } x \in K \setminus (C_{l,m,\delta} \cup B_\delta a^{r_l m});$$

(2.4) and (2.7) imply that

$$\varphi_{l,m}(x) < \frac{\delta}{|f(x)|} < \frac{\delta}{1-\delta} < \varepsilon \quad \text{for } x \in K \setminus (D_{l,m,\delta} \cup A_\delta).$$

By (2.6) and (2.8), for $x \in K \setminus (C_{l,m,\delta} \cup a^{(r_l - r_s)m} F_{s,m,\delta})$ we have

$$\begin{aligned}
\frac{w_l(x) \dots w_l(xa^{-(r_l m-1)})}{w_s(xa^{1-r_l m}) \dots w_s(xa^{r_s m - r_l m})} &= \frac{w_l(x) \dots w_l(xa^{-(r_l m-1)})|f(xa^{-r_l m})|}{w_s(xa^{1-r_l m}) \dots w_s(xa^{r_s m - r_l m})|f(xa^{-r_l m})|} \\
&= \frac{\widetilde{\varphi}_{l,m}(x)^{-1}|f(xa^{-r_l m})|}{\widetilde{\varphi}_{s,m}(xa^{(r_s - r_l)m})^{-1}|f(xa^{(r_s-r_l)m}a^{-r_s m})|} \\
&> \frac{1-\delta}{\delta} > \frac{1}{\varepsilon} \quad \text{if } 1 \leqslant s < l \leqslant N.
\end{aligned}$$

Similarly, for $x \in K \setminus (C_{s,m,\delta} \cup a^{(r_s - r_l)m} F_{l,m,\delta})$ we have

$$\frac{w_s(x) \dots w_s(xa^{1-r_s m})}{w_l(xa^{1-r_s m}) \dots w_l(xa^{r_l m - r_s m})} > \frac{1}{\varepsilon} \quad \text{if } 1 \leqslant s < l \leqslant N.$$

Let

$$\begin{aligned}
\widetilde{B}_{m,\delta} &= B_\delta a^{r_1 m} \cup \ldots \cup B_\delta a^{r_N m}, \\
\widetilde{C}_{m,\delta} &= C_{1,m,\delta} \cup \ldots \cup C_{N,m,\delta}, \\
\widetilde{D}_{m,\delta} &= D_{1,m,\delta} \cup \ldots \cup D_{N,m,\delta}, \\
\widetilde{F}_{m,\delta} &= \bigcup_{1 \leqslant s < l \leqslant N} a^{(r_l - r_s)m} F_{s,m,\delta}, \\
\widetilde{G}_{m,\delta} &= \bigcup_{1 \leqslant s < l \leqslant N} a^{(r_s - r_l)m} F_{l,m,\delta}.
\end{aligned}$$

Now, let $H_{m,\delta} = A_\delta \cup \widetilde{B}_{m,\delta} \cup \widetilde{C}_{m,\delta} \cup \widetilde{D}_{m,\delta} \cup \widetilde{F}_{m,\delta} \cup \widetilde{G}_{m,\delta}$, $E_{m,\delta} = K \setminus H_{(m,\delta)}$. Then $\lambda(H_{m,\delta}) < (1+N)^2 \delta^p < (1+N)^2 \varepsilon^p$ and

$$\|\varphi_{l,m}|_{E_{m,\delta}}\|_\infty < \varepsilon, \qquad \|\widetilde{\varphi}_{l,m}|_{E_{m,\delta}}\|_\infty < \varepsilon, \tag{2.9}$$

$$\left\| \frac{\prod_{t=1}^{r_s m} w_s * \delta_{a^{-1}}^{t - r_l m}}{\prod_{t=0}^{r_l m - 1} w_l * \delta_a^t}\Big|_{E_{m,\delta}} \right\|_\infty < \varepsilon, \qquad \left\| \frac{\prod_{t=1}^{r_l m} w_l * \delta_{a^{-1}}^{t - r_s m}}{\prod_{t=0}^{r_s m - 1} w_s * \delta_a^t}\Big|_{E_{m,\delta}} \right\|_\infty < \varepsilon. \tag{2.10}$$

For $k = 1, 2, \ldots$, taking $\varepsilon = 1/k$ in the above arguments and denoting m by n_k, $E_{m,\delta}$ by E_k, we get a sequence $\{n_k\}$ of positive integers and a sequence $\{E_k\}$ of subsets in K such that $\lambda(K) = \lim_{k \to \infty} \lambda(E_k)$ and (2.1) and (2.2) hold. $\square$

By Theorem 2.7 in [6], condition (ii) of Theorem 2.1 also implies that the operators $T_1, \ldots, T_N$ satisfy the d-hypercyclicity criterion. Indeed, for each $r \in \mathbb{N}$, if one considers nonempty open sets

$$V_{0,j}, V_{1,j}, \ldots, V_{N,j}, \quad j = 1, \ldots, r$$

in $L^p(G)$ and picks $f_{0,j},\ g_{1,j}, \ldots, g_{N,j} \in C_c(G)$ with conditions $f_{0,j} \in V_{0,j}$, $g_{1,j} \in V_{1,j}$, …, $g_{N,j} \in V_{N,j}$, then the same arguments as in the proof of (ii) $\Rightarrow$ (i) in Theorem 2.1 can be applied to these functions to obtain r sequences $(v_{1,k}), \ldots, (v_{r,k})$ in $L^p(G)$ satisfying

$$\lim_{k\to\infty} v_{j,k} = f_{0,j} \text{ and } \lim_{k\to\infty} T_{a,w_l}^{r_l n_k} v_{j,k} = g_{l,j} \quad \text{for } 1 \leqslant l \leqslant N,\ 1 \leqslant j \leqslant r,$$

yielding

$$V_{0,j} \cap T_{a,w_1}^{-r_1 n_k}(V_{1,j}) \cap \ldots \cap T_{a,w_N}^{-r_N n_k}(V_{N,j}) \neq \emptyset \quad \text{for some } k.$$

Hence we can draw the following result.

Corollary 2.2. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. For each $1 \leqslant l \leqslant N$, let $w_l\colon G \to (0, \infty)$ be a weight on G and T_{a,w_l} a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ *are densely d-hypercyclic.*

(ii) $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ *satisfy the d-hypercyclicity criterion.*

Using arguments similar to those in the proof of Theorem 2.1, we can also characterize d-topological mixing powers of weighted translations for nondiscrete groups.

Corollary 2.3. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. For each $1 \leqslant l \leqslant N$, let $w_l\colon G \to (0, \infty)$ be a weight on G and T_{a,w_l} a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ *are d-mixing.*

(ii) *For $1 \leqslant l \leqslant N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and the sequences*

$$\varphi_{l,k} := \prod_{s=1}^{r_l k} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,k} := \left(\prod_{s=0}^{r_l k - 1} w_l * \delta_a^s\right)^{-1},$$

satisfy

$$\lim_{k\to\infty}\|\varphi_{l,k}|_{E_k}\|_\infty = \lim_{k\to\infty}\|\widetilde{\varphi}_{l,k}|_{E_k}\|_\infty = 0, \tag{2.11}$$

and, if $1 \leqslant s < l < N$,

$$\lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_s k} w_s * \delta_{a^{-1}}^{t-r_l k}}{\prod_{t=0}^{r_l k-1} w_l * \delta_a^t}\Big|_{E_k}\right\|_\infty = \lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_l k} w_l * \delta_{a^{-1}}^{t-r_s k}}{\prod_{t=0}^{r_s k-1} w_s * \delta_a^t}\Big|_{E_k}\right\|_\infty = 0. \tag{2.12}$$

If G is discrete, then $E_m = K$ in the proof of Theorem 2.1. Hence we have the following characterization of disjoint hypercyclic powers of weighted translation operators on discrete groups.

Corollary 2.4. *Let G be a discrete group, and let a be a torsion free element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. For each $1 \leqslant l \leqslant N$, let $w_l \colon G \to (0,\infty)$ be a weight on G and T_{a,w_l} a weighted translation on $l^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ *are densely d-hypercyclic.*

(ii) *For $1 \leqslant l \leqslant N$ and each finite subset $K \subseteq G$ for the sequences*

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n-1} w_l * \delta_a^s\right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty}\|\varphi_{l,n_k}|_K\|_\infty = \lim_{k\to\infty}\|\widetilde{\varphi}_{l,n_k}|_K\|_\infty = 0, \tag{2.13}$$

and, if $1 \leqslant s < l < N$,

$$\lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k-1} w_l * \delta_a^t}\Big|_K\right\|_\infty = \lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k-1} w_s * \delta_a^t}\Big|_K\right\|_\infty = 0. \tag{2.14}$$

Example. Let $G = \mathbb{Z}$, $a = -1$. For each $1 \leqslant l \leqslant N$, we consider the weighted translation T_l on $l^2(\mathbb{Z})$ defined by $T_l = T_{-1,w_l*\delta_{-1}}$, where (w_l) is a sequence of positive weights. Then T_l is a bilateral weighted shift on $l^2(\mathbb{Z})$, that is, $T_l e_j = w_{l,j}e_{j-1}$ with $w_{l,j} = w_l(j)$ for each l. Here $(e_j)_{j\in\mathbb{Z}}$ is the canonical basis of $l^2(\mathbb{Z})$. Let $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. Next, by Corollary 2.4, the operators $T_1^{r_1}, \dots, T_N^{r_N}$ are densely d-hypercyclic if and only if given $\varepsilon > 0$ and $q \in \mathbb{N}$, there exists $m \in \mathbb{N}$ such that for $|j| \leqslant q$ we have

$$\tag{2.15} \begin{cases} \Big|\prod\limits_{i=j+1}^{j+r_l m} w_l(i)\Big| > \dfrac{1}{\varepsilon}, \\ \Big|\prod\limits_{i=j-r_l m+1}^{j} w_l(i)\Big| < \varepsilon, \end{cases} \qquad 1 \leqslant l \leqslant N,$$

and

$$\tag{2.16} \begin{cases} \Big|\prod\limits_{i=j+1}^{j+r_l m} w_l(i)\Big| > \dfrac{1}{\varepsilon}\Big|\prod\limits_{i=j+(r_l-r_s)m+1}^{j+r_l m} w_s(i)\Big|, \\ \Big|\prod\limits_{i=j-(r_l-r_s)m+1}^{j+r_s m} w_l(i)\Big| < \varepsilon\Big|\prod\limits_{i=j+1}^{j+r_s m} w_s(i)\Big|, \end{cases} \qquad 1 \leqslant s < l \leqslant N,$$

which are the same as in [6], Theorem 4.7.

3. Disjoint supercyclic powers of weighted translations

It is well known that a complex Banach space admits a supercyclic operator if it is one dimensional or infinite-dimensional and separable. Chen in [7] characterized supercyclic weighted translation operators on the Lebesgue space $L^p(G)$ in terms of the weight. Inspired by his work, in this section we will give sufficient and necessary conditions for disjoint supercyclic powers of weighted translations generated by aperiodic elements on groups.

Theorem 3.1. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \dots, N$. For each $1 \leqslant l \leqslant N$, let $w_l\colon G \to (0, \infty)$ be a weight on G and T_{a,w_l} a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) *$T_{a,w_1}^{r_1}, \dots, T_{a,w_N}^{r_N}$ are densely d-supercyclic.*

(ii) *For $1 \leqslant l \leqslant N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K and there exist sequences $(\alpha_{l,n})_n \subseteq \mathbb{C} \setminus \{0\}$ such that $\lambda(K) = \lim\limits_{k\to\infty} \lambda(E_k)$ and for the sequences*

$$\varphi_{l,n} := |\alpha_{l,n}| \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,n} := \Bigg(|\alpha_{l,n}| \prod_{s=0}^{r_l n - 1} w_l * \delta_a^s\Bigg)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\tag{3.1} \lim_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = \lim_{k\to\infty} \|\widetilde{\varphi}_{l,n_k}|_{E_k}\|_\infty = 0,$$

and, if $1 \leqslant s < l < N$,

$$\lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k-1} w_l * \delta_a^t}\Big|_{E_k}\right\|_\infty = \lim_{k\to\infty}\left\|\frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k-1} w_s * \delta_a^t}\Big|_{E_k}\right\|_\infty = 0. \tag{3.2}$$

Proof. (i) $\Rightarrow$ (ii). Let $T_{a,w_1}^{r_1},\ldots,T_{a,w_N}^{r_N}$ be densely d-supercyclic. Let $K\subseteq G$ be a compact set with $\lambda(K)>0$. Let $\varepsilon>0$. Due to aperiodicity of a, there exists $M\in\mathbb{N}$ such that $K\cap Ka^{\pm n}=\emptyset$ for all $n>M$. Let $\chi_K\in L^p(G)$ be the characteristic function of K. Choose $0<\delta<\varepsilon/(1+\varepsilon)$. By assumption, there exists a d-supercyclic vector $f\in L^p(G)$ and some $m>M$ and $\alpha\in\mathbb{C}\setminus\{0\}$ such that for $1\leqslant l\leqslant N$,

$$\|f-\chi_K\|_p<\delta^2 \quad\text{and}\quad \|\alpha T_{a,w_l}^{r_l m}f-\chi_K\|_p<\delta^2. \tag{3.3}$$

The rest is similar to the proof of (i) $\Rightarrow$ (ii) in Theorem 2.1, so we omit the details.

(ii) $\Rightarrow$ (i). A simple Baire category argument and Birkhoff transitivity theorem show that $T_{a,w_1}^{r_1},\ldots,T_{a,w_N}^{r_N}$ are densely d-supercyclic provided for every nonempty open subsets $V_0,\ldots,V_N$ of $L^p(G)$, there exist $m\in\mathbb{N}$ and $\lambda_m\in\mathbb{C}\setminus\{0\}$ such that $\emptyset\neq V_0\cap\lambda_m^{-1}T_{a,w_1}^{-r_1 m}(V_1)\cap\ldots\cap\lambda_m^{-1}T_{a,w_N}^{-r_N m}(V_N)$.

Let $V_0,\ldots,V_N$ be nonempty open subsets of $L^p(G)$. Since the space $C_c(G)$ of continuous functions on G with compact support is dense in $L^p(G)$, we can pick $f,g_1,\ldots,g_N\in C_c(G)$ with $f\in V_0$, $g_1\in V_1,\ldots,g_N\in V_N$. Let K be the union of the compact supports of f, $g_1,\ldots,g_N$ and let $\chi_K\in L^p(G)$ be the characteristic function of K. For $1\leqslant l\leqslant N$, let $E_k\subseteq K$ and let there exist an increasing subsequence $(n_k)\subseteq\mathbb{N}$ satisfying conditions (3.1) and (3.2).

By aperiodicity of a, there exists $M\in\mathbb{N}$ such that $K\cap Ka^{\pm n}=\emptyset$ for all $n>M$. Similarly to the proof of Theorem (2.1), for $1\leqslant l\leqslant N$ we define self-maps S_{a,w_l} on the subspace $L_c^p(G)$ of functions in $L^p(G)$ with compact support by

$$S_{a,w_l}(h)=\frac{h}{w_l}*\delta_{a^{-1}},\quad h\in L_c^p(G)$$

such that

$$T_{a,w_l}^{r_l n_k}S_{a,w_l}^{r_l n_k}(h)=h,\quad h\in L_c^p(G).$$

A similar calculation to that used in Theorem 2.1 will show

$$\lim_{k\to\infty}\|\alpha_{l,n_k}T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p=0;$$
$$\lim_{k\to\infty}\left\|\frac{1}{\alpha_{l,n_k}}S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\right\|_p=0;$$
$$\lim_{k\to\infty}\|T_{a,w_l}^{r_l n_k}S_{a,w_s}^{r_s n_k}(g_s\chi_{E_k})\|_p=0;$$
$$\lim_{k\to\infty}\|T_{a,w_s}^{r_s n_k}S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\|_p=0.$$

Hence, we have

$$\lim_{k\to\infty} \|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p \|S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\|_p = 0,$$

and

$$\lim_{k\to\infty} \left\| \sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) - g_l\chi_{E_k} \right\|_p = 0.$$

By passing to a subsequence if necessary, we may assume that for $1 \leqslant l \leqslant N$,

$$\|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p \|S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k})\|_p < \frac{1}{4k^2} \tag{3.4}$$

and

$$\left\| \sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) - g_l\chi_{E_k} \right\|_p < \frac{1}{2k}. \tag{3.5}$$

Now, let

$$v_k = f\chi_{E_k} + \frac{1}{\alpha_{n_k}} \sum_{i=1}^{N} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) \in L^p(G),$$

where $\alpha_{n_k} := 2k \left\| \sum_{i=1}^{N} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) \right\|_p$. Then for $1 \leqslant l \leqslant N$,

$$\|v_k - f\|_p \leqslant \|f\|_\infty \lambda(K \setminus E_k)^{1/p} + \frac{1}{2k},$$

and

$$\begin{aligned}
\|\alpha_{n_k} T_{a,w_l}^{r_l n_k} v_k - g_l\|_p &\leqslant \|\alpha_{n_k} T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p \\
&\quad + \left\| \sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) - g_l \right\|_p \\
&\leqslant \|\alpha_{n_k} T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\|_p + \|g_l\|_\infty \lambda(K \setminus E_k)^{1/p} \\
&\quad + \left\| \sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i\chi_{E_k}) - g_l\chi_{E_k} \right\|_p \\
&\leqslant \frac{1}{2k} + \|g_l\|_\infty \lambda(K \setminus E_k)^{1/p} + \frac{1}{2k}.
\end{aligned}$$

Hence, $\lim_{k\to\infty} v_k = f$ and $\lim_{k\to\infty} \alpha_{n_k} T_{a,w_l}^{r_l n_k} v_k = g_l$, which imply

$$V_0 \cap \alpha_{n_k}^{-1} T_{a,w_1}^{-r_1 n_k}(V_1) \cap \ldots \cap \alpha_{n_k}^{-1} T_{a,w_N}^{-r_N n_k}(V_N) \neq \emptyset \quad \text{for some } k.$$

Therefore, $T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ are densely d-supercyclic. □

Remark 3.2. By Corollary 2.5 in [7], it is easily shown that the condition (ii) in Theorem 3.1 holds if and only if for $1 \leqslant l \leqslant N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and for the sequences

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n-1} w_l * \delta_a^s \right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k} \, \widetilde{\varphi}_{l,n_k}|_{E_k}\|_\infty = 0 \tag{3.6}$$

and (3.2) holds.

If G is discrete, by the proof of Theorem 3.1 we have the following characterization of disjoint supercyclic powers of weighted translation operators on discrete groups.

Corollary 3.3. *Let G be a discrete group, and let a be a torsion free element in G. Let $1 \leqslant p < \infty$ and $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $N \geqslant 2$, $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. For each $1 \leqslant l \leqslant N$, let $w_l \colon G \to (0, \infty)$ be a weight on G and T_{a,w_l} a weighted translation on $l^p(G)$. The following conditions are equivalent:*

(i) *$T_{a,w_1}^{r_1}, \ldots, T_{a,w_N}^{r_N}$ are densely d-supercyclic.*

(ii) *For $1 \leqslant l \leqslant N$ and each finite subset $K \subseteq G$ for the sequences*

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \widetilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n-1} w_l * \delta_a^s \right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k} \, \widetilde{\varphi}_{l,n_k}|_K\|_\infty = 0 \tag{3.7}$$

and, if $1 \leqslant s < l < N$,

$$\lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k - 1} w_l * \delta_a^t} \bigg|_K \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k - 1} w_s * \delta_a^t} \bigg|_K \right\|_\infty = 0. \tag{3.8}$$

Example. Let $G = \mathbb{Z}$, $a = -1$. For each $1 \leqslant l \leqslant N$ we consider weighted translation T_l on $l^2(\mathbb{Z})$, defined by $T_l = T_{-1, w_l * \delta_{-1}}$, where (w_l) is a positive weight. Then T_l is a bilateral weighted shift on $l^2(\mathbb{Z})$, that is, $T_l e_j = w_{l,j} e_{j-1}$ with $w_{l,j} = w_l(j)$ for each l. Here $(e_j)_{j\in\mathbb{Z}}$ is the canonical basis of $l^2(\mathbb{Z})$. Let $1 \leqslant r_1 < r_2 < \ldots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \ldots, N$. Next, by Corollary 3.3, the operators $T_1^{r_1}, \ldots, T_N^{r_N}$ are densely d-supercyclic if and only if given $\varepsilon > 0$ and $q \in \mathbb{N}$, there exists $m \in \mathbb{N}$ so that for $|j| \leqslant q$, $|k| \leqslant q$ and $1 \leqslant s, l \leqslant N$ we have

$$\Big|\prod_{i=j-r_l m+1}^{j} w_l(i)\Big| < \varepsilon\Big|\prod_{i=k+1}^{k+r_s m} w_s(i)\Big|, \quad 1 \leqslant l, s \leqslant N, \tag{3.9}$$

and

$$\begin{cases} \Big|\prod\limits_{i=j+1}^{j+r_l m} w_l(i)\Big| > \dfrac{1}{\varepsilon}\Big|\prod\limits_{i=j+(r_l-r_s)m+1}^{j+r_l m} w_s(i)\Big|, \\ \Big|\prod\limits_{i=j-(r_l-r_s)m+1}^{j+r_s m} w_l(i)\Big| < \varepsilon\Big|\prod\limits_{i=j+1}^{j+r_s m} w_s(i)\Big|, \end{cases} \quad 1 \leqslant s < l \leqslant N, \tag{3.10}$$

which are the same as in [15], Theorem 4.2.1.

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